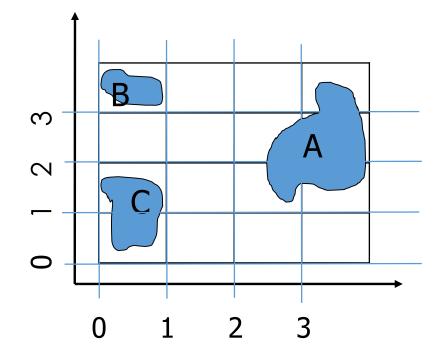
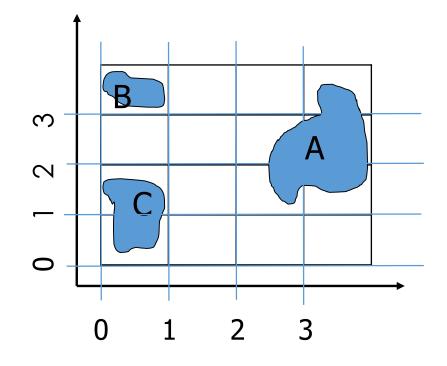
# Introduction to Spatial Computing CSE 555

Spatial Indexing Techniques for Secondary Memory

Some slides adapted from Spatial Databases: A Tour by Shashi Shekhar Prentice Hall (2003)



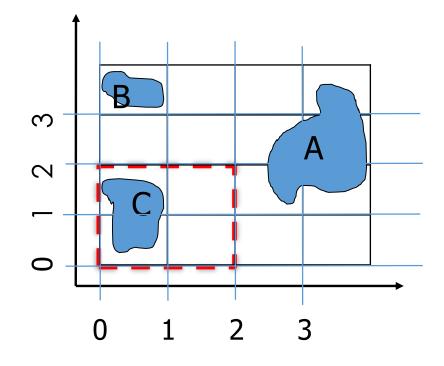
- Goal: Store spatial objects A,B and C in storage system such that following queries can be executed efficiently.
- Point Queries:
- Range Queries:
- Nearest Neighbor Queries
- Spatial Joins:



 Goal: Store spatial objects A,B and C in storage system such that following queries can be executed efficiently.

#### Point Queries:

- Given an object search if it exists in the database or not
- Example: Return the spatial object located at (3,2)
- Range Queries:
- Nearest Neighbor Queries
- Spatial Joins:

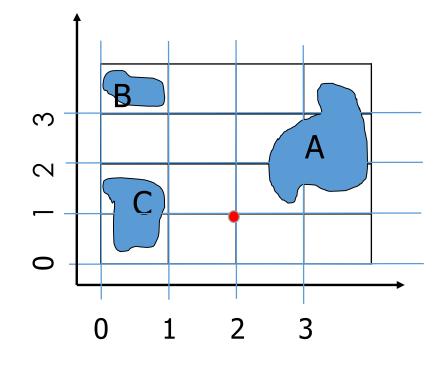


 Goal: Store spatial objects A,B and C in storage system such that following queries can be executed efficiently.

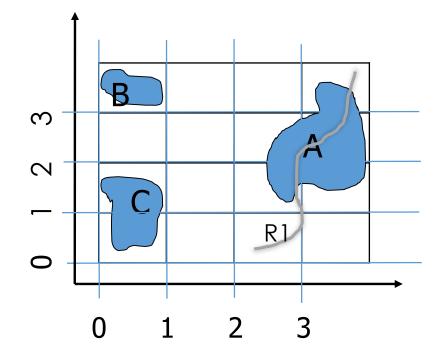
#### Point Queries:

#### Range Queries:

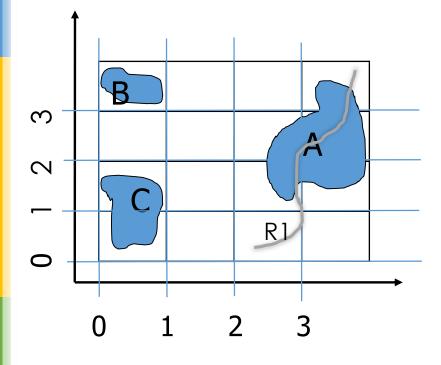
- Return the objects which lie within the defined range of x and y
- Example: return objects which lie in the rectangle defined by 0<x<2 and 0<y<2</li>
- Nearest Neighbor Queries
- Spatial Joins:



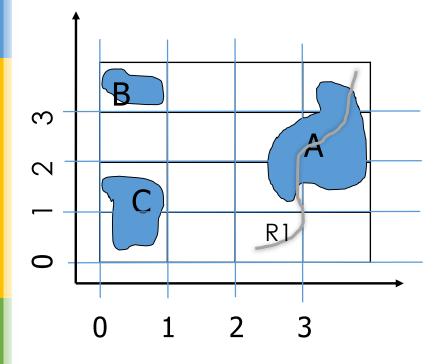
- Goal: Store spatial objects A,B and C in storage system such that following queries can be executed efficiently.
- Point Queries:
- Range Queries:
- Nearest Neighbor Queries
  - Find the nearest spatial object (or k nearest spatial objects) of the point (2,1)
- Spatial Joins:



- Goal: Store spatial objects A,B and C in storage system such that following queries can be executed efficiently.
- Point Queries:
- Range Queries:
- Nearest Neighbor Queries
- Spatial Joins:
  - Find the spatial objects which intersect the object R1

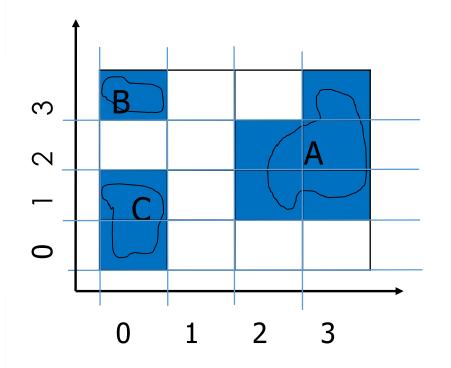


- Had these objects been a 1-dimensional in nature, e.g., real numbers, strings etc.
- A simple B+ tree would be constructed over these.
- Can easily get O(log n) complexity for all the queries (except the join query) mentioned in the previous slides.

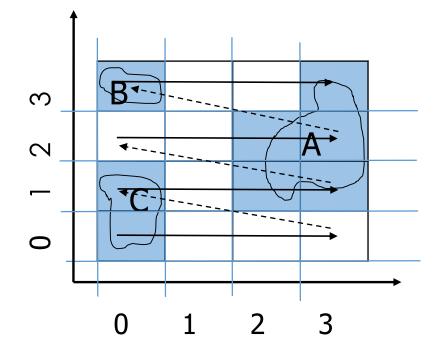


- How to get ordering in 2-Dimensions?
- Once we get ordering we can try B+ tree again for spatial objects.

### Towards Getting an Order Basics



- Approximate objects with cells.
- Helps in getting a continuous space to work with easer to handle.
- Would have to map back whenever necessary (for the queries and results).

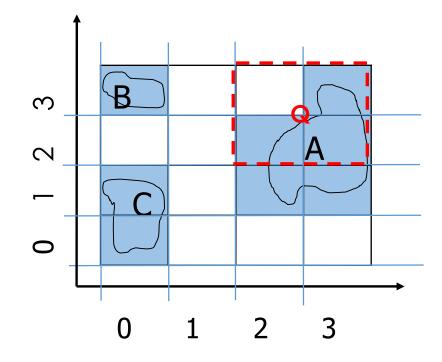


#### **First Attempt**

Order on Y then X: (0,0) (1,0) (2,0)
 (3,0) (0,1) (1,1) (2,1) (3,1) (0,2) (1,2)
 (2,2) (3,2) (0,3) (1,3) (2,3) (3,3)

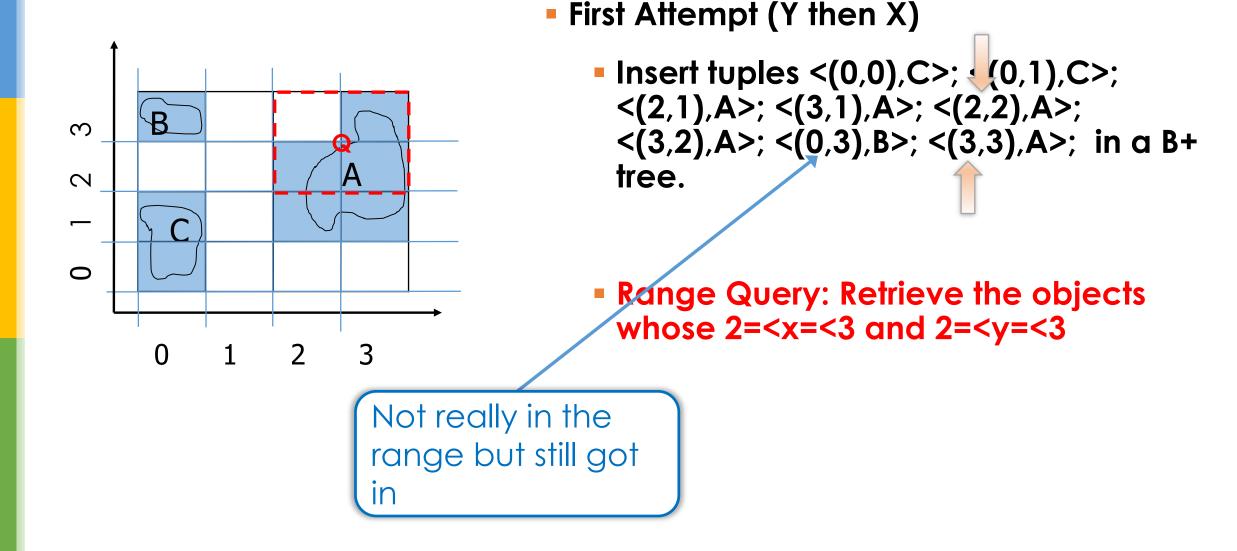
 **First Attempt** 

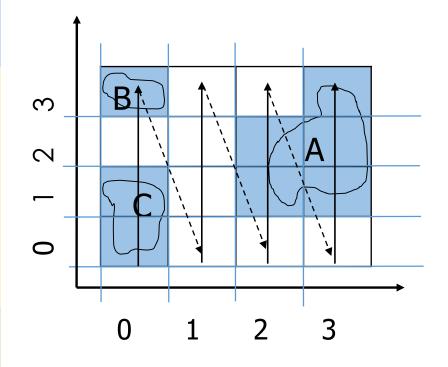
- Order on Y then X: (0,0) (1,0) (2,0) (3,0)
  (0,1) (1,1) (2,1) (3,1) (0,2) (1,2) (2,2)
  (3,2) (0,3) (1,3) (2,3) (3,3)
- Insert tuples <(0,0),C>; <(0,1),C>; <(2,1),A>; <(3,1),A>; <(2,2),A>; <(3,2),A>; <(0,3),B>; <(3,3),A>; in a B+ tree.
- These would be order of leaves in the B+ tree



#### First Attempt (Y then X)

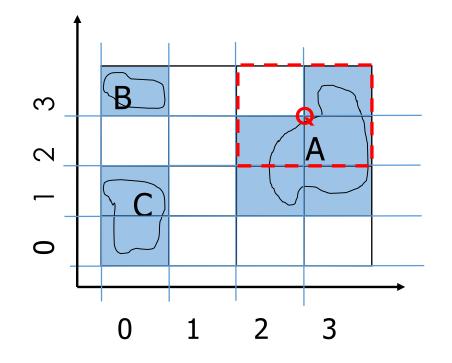
- Insert tuples <(0,0),C>; <(0,1),C>;<(2,1),A>; <(3,1),A>; <(2,2),A>;<(3,2),A>; <(0,3),B>; <(3,3),A>; in aB+ tree.
- Range Query: Retrieve the objects whose 2=<x=<3 and 2=<y=<3</p>





Second Attempt (X then Y) Order on X then Y:

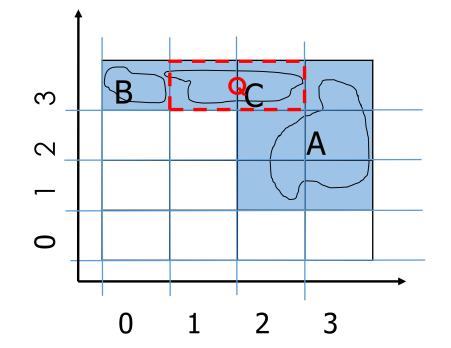
 $\begin{array}{c} \textbf{(0,0)} \ \textbf{(0,1)} \ \textbf{(0,2)} \ \textbf{(0,3)} \ \textbf{(1,0)} \ \textbf{(1,1)} \ \textbf{(1,2)} \\ \textbf{(1,3)} \ \textbf{(2,0)} \ \textbf{(2,1)} \ \textbf{(2,2)} \ \textbf{(2,3)} \ \textbf{(3,0)} \ \textbf{(3,1)} \\ \textbf{(3,2)} \ \textbf{(3,3)} \end{array}$ 



Second Attempt (X then Y) Order on X then Y: (0,0) (0,1) (0,2) (0,3) (1,0) (1,1) (1,2) (1,3) (2,0) (2,1) (2,2) (2,3) (3,0) (3,1) (3,2) (3,3)

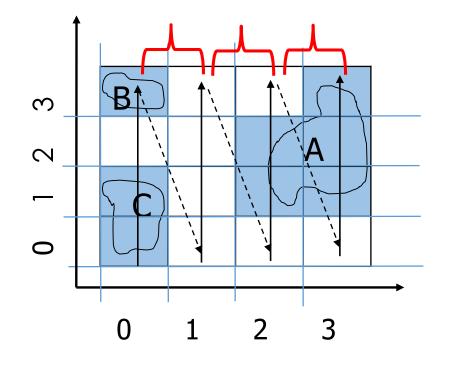
Range Query 2<x<3 & 2<y<3: Little better this time

#### How about in this scenario?



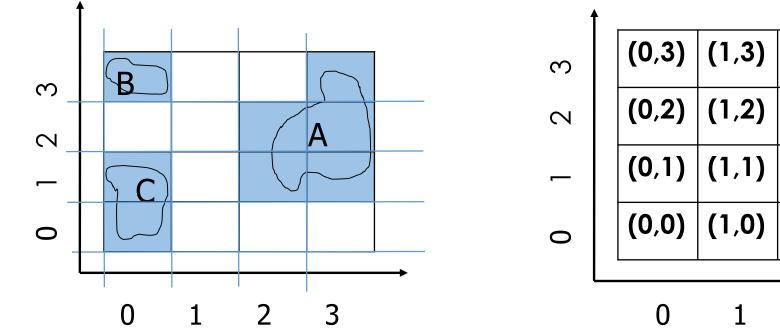
Range Query: Retrieve all objects in this range 1=<x=<2 & y=3?

Neighboring Cells but far apart in the ordering



 Problem with these orderings: Cells which are close to each other might get spread out and occupy places quite far from each other.

- Need a ordering which can preserve spatial locality in both x and y directions as much as possible!
- Cannot get 100%

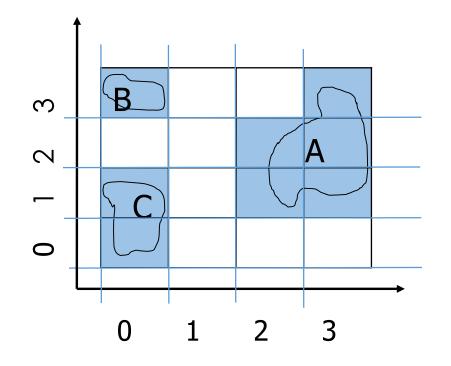


(1,2) (2,2) (3,2) (1,1) (2,1) (3,1) (1,0) (2,0) (3,0)

(2,3) (3,3)

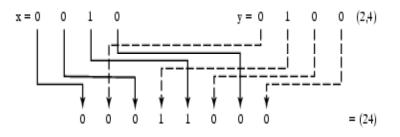
3

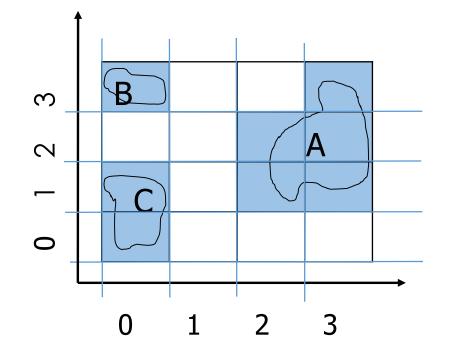
2



4	00	01	10	11
3	11	11	11	11
	00	01	10	11
2	10	10	10	10
	00	01	10	11
-	01	01	01	01
	00	01	10	11
0	00	00	00	00
I				
	0	1	2	3

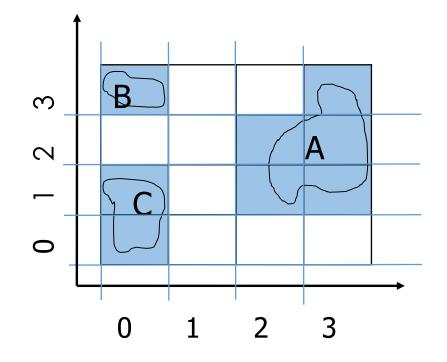
Write the X and Y coordinates in Binary Form





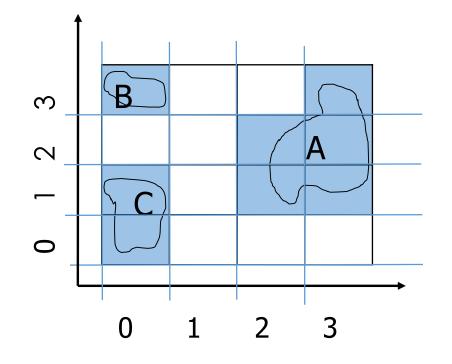
	1			
с	0101	0111	1101	1111
2	0100	0110	1100	1110
-	0001	0011	1001	1011
0	0000	0010	1000	1010
	0	1	2	3
		_		

Interleave them to create one string



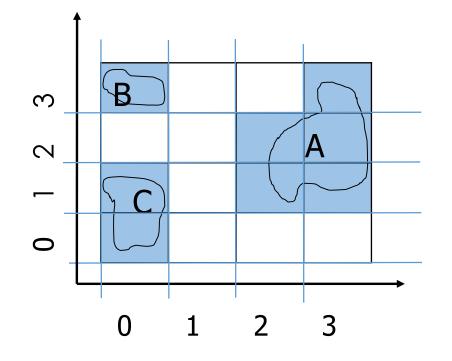
1	•			
S	5	7	13	15
2	4	6	12	14
1	1	3	9	11
0	0	2	8	10
	0	1	2	3

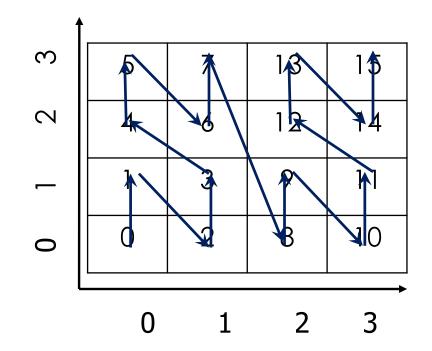
Convert the bit strings to its corresponding decimal



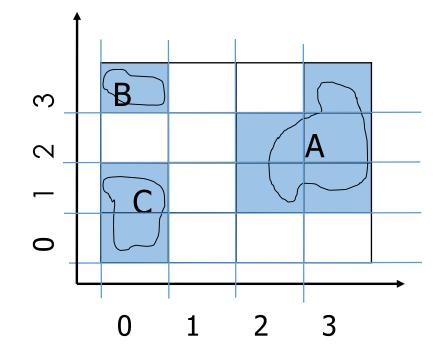
4				
Э	5	7	13	15
7	4	6	12	14
1	1	3	9	11
0	0	2	8	10
				$\rightarrow$
	0	T	2	3

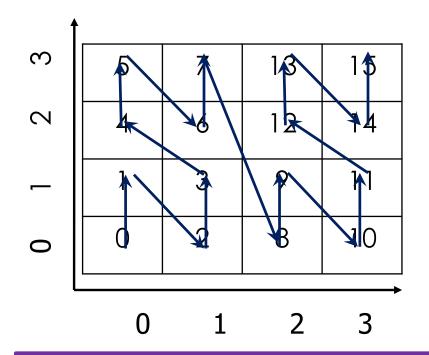
This is the order of cells from this process





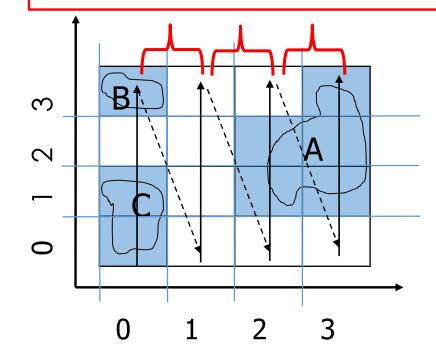
This is the order of cells from this process



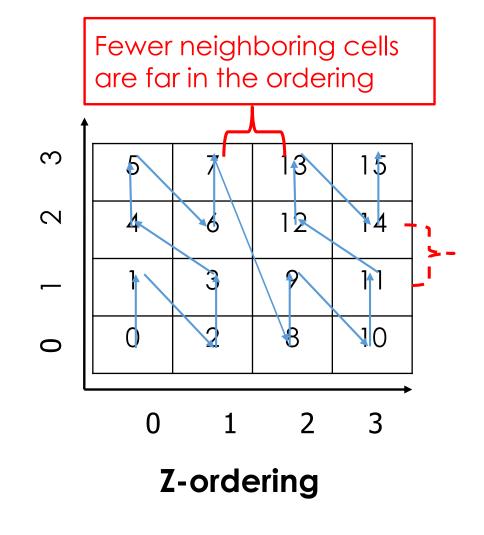


Visually its looks like we have Zs on our map. Hence the name Z-order curve!!

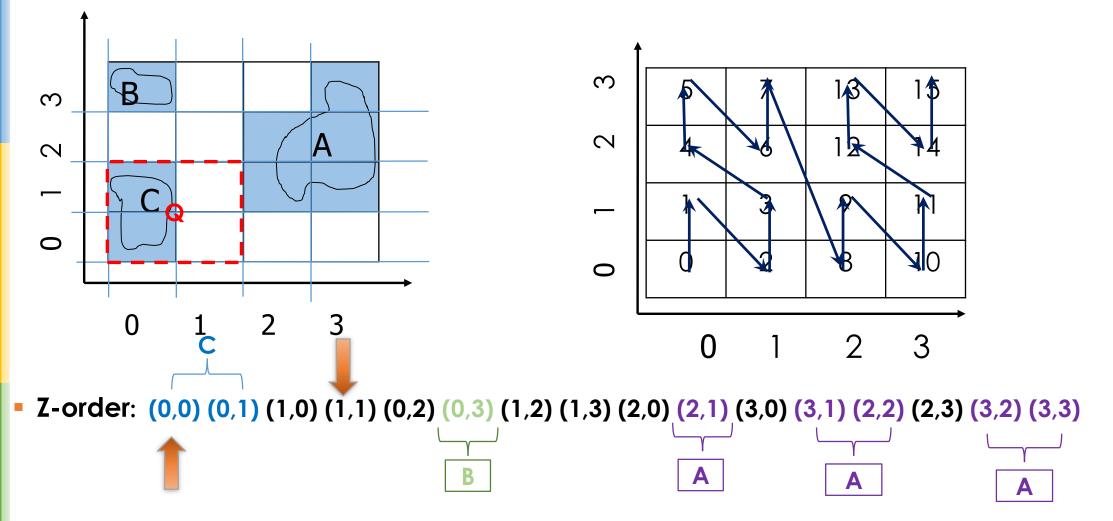
Many neighboring cells thrown far apart in the ordering



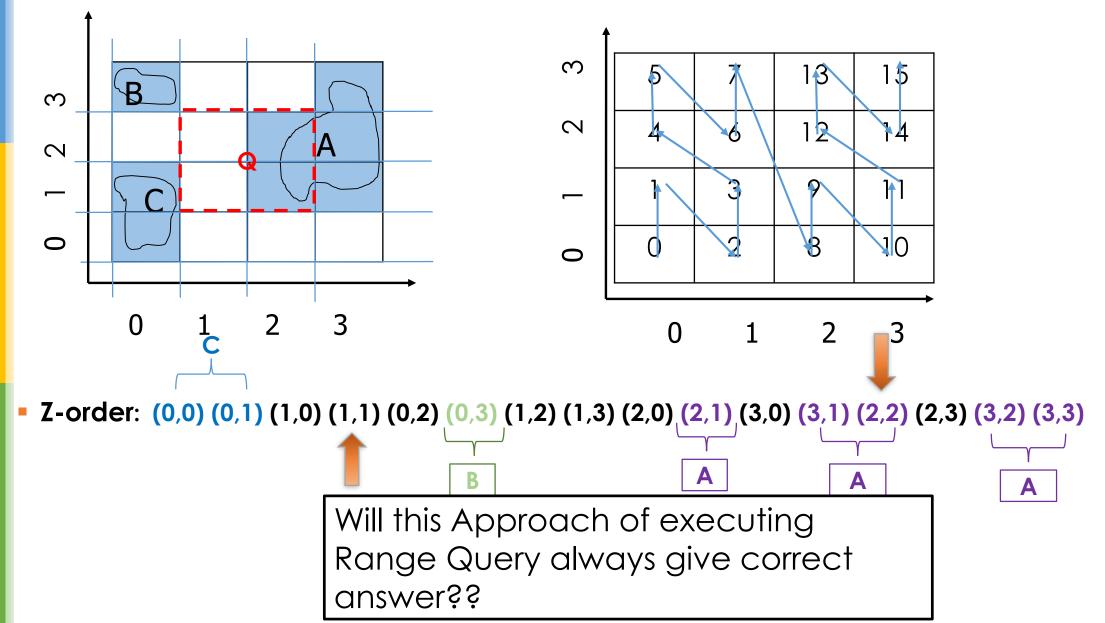
Ordering: X followed by Y



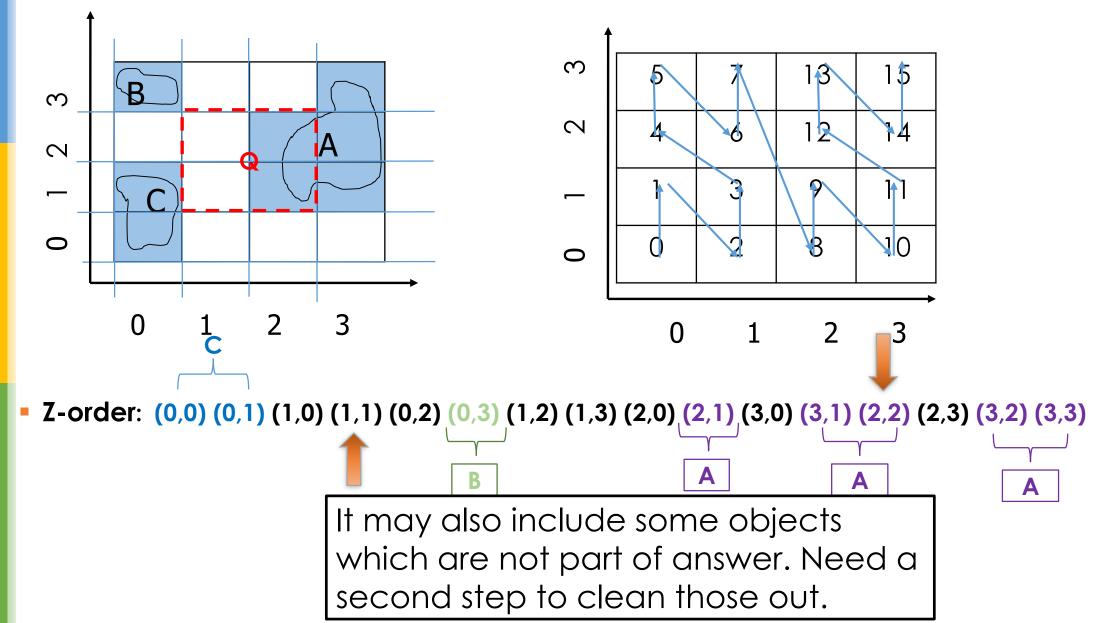
#### Z-Order curve: Range Query



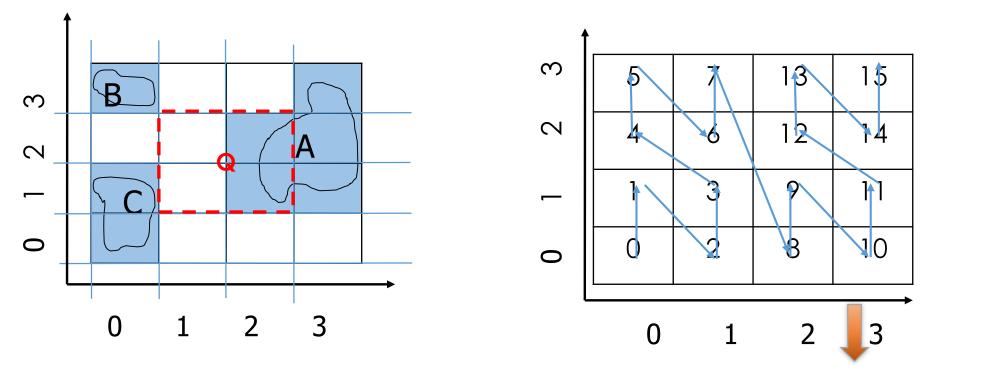
Z-Order curve: Range Query



Z-Order curve: Range Query



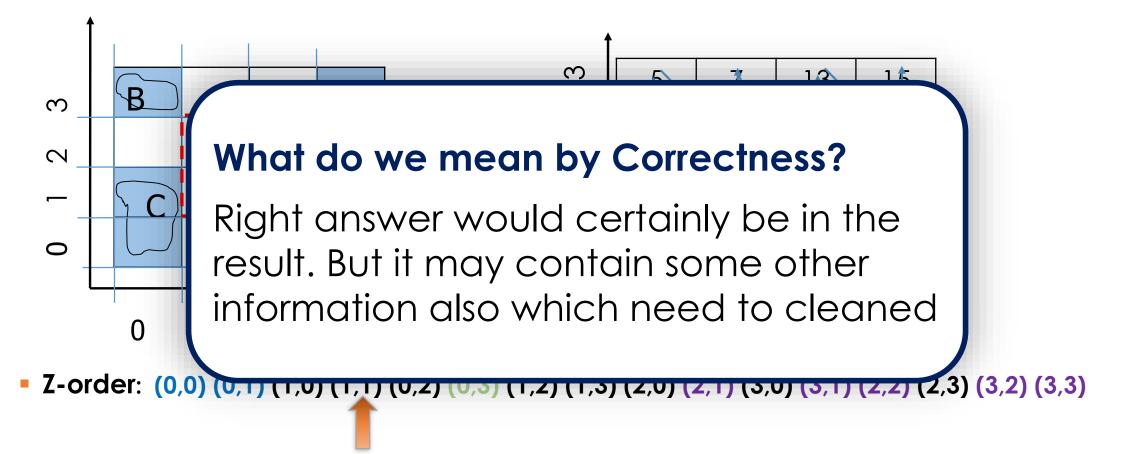
Consider again our previous example:



Z-order: (0,0) (0,1) (1,0) (1,1) (0,2) (0,3) (1,2) (1,3) (2,0) (2,1) (3,0) (3,1) (2,2) (2,3) (3,2) (3,3)

Retrieved all records within this range and cross checked the result.

Consider again our previous example:



Retrieved all records within this range and cross checked the result.

#### Proof Sketch:

- Z-order: (0,0) (0,1) (1,0) (1,1) (0,2) (0,3) (1,2) (1,3) (2,0) (2,1) (3,0) (3,1) (2,2) (2,3) (3,2) (3,3)
- Retrieved all records within this range and cross checked the result.
- For this approach to be correct we need to prove that all the cells which are in the query rectangle of (1,1) and (2,2) are between 4 and 9.

#### Proof Sketch:

#### Without loss of generalization let:

- LL = (xmin, ymin) is the lower left of the query rectangle
- UR = (xmax, ymax) is the upper right of the query rectangle.
- Then we need to prove that all the cells with (xmin < x < xmax) and (ymin < y < ymax) will have their Z-values between z-values of LL and UR.</p>

#### **Proof Sketch:**

Take two cell coordinates numbers: (x1,y1) and (x2, y2)

#### Case I: x2 > x1 and y1 = y2

- If x2 is greater than x1 that it will have "1" in at least one higher position in binary form
- Which means it will get "1" in at least one higher position in its z-value.
- Implies that it will have a higher z-value.

#### **Proof Sketch:**

#### Case II: y2 > y1 and x1 = x2

- If y2 is greater than y1 that it will have "1" in at least one higher position in binary form
- Which means it will get "1" in at least one higher position in its z-value
- Implies it will have a higher z-value.

#### **Proof Sketch:**

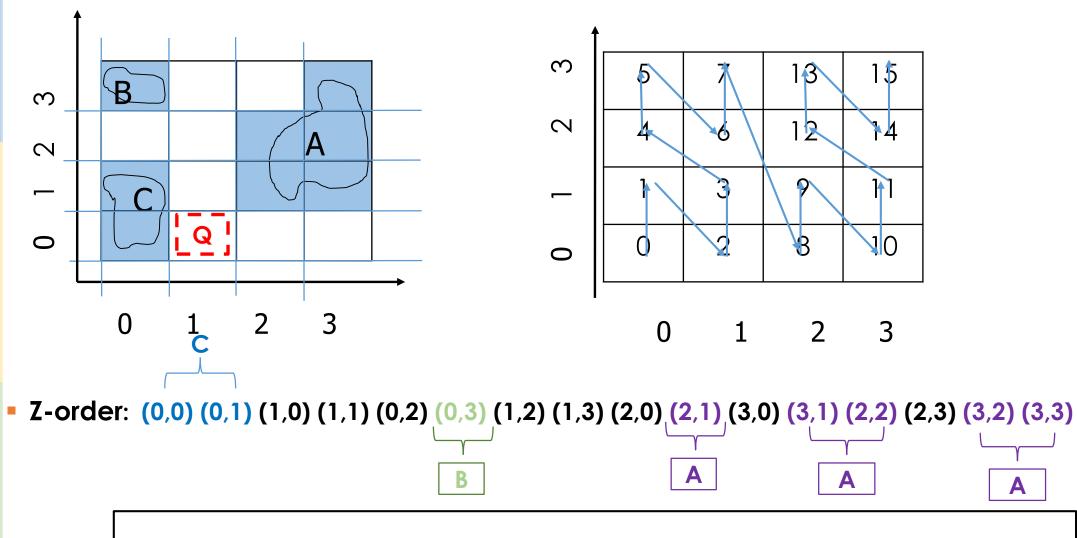
#### Case III: x2 > x1 and y2 > y1

- Similar argument of getting "1" in at least one higher position in its z-value
- Implies it will have a higher z-value

#### **Proof Sketch:**

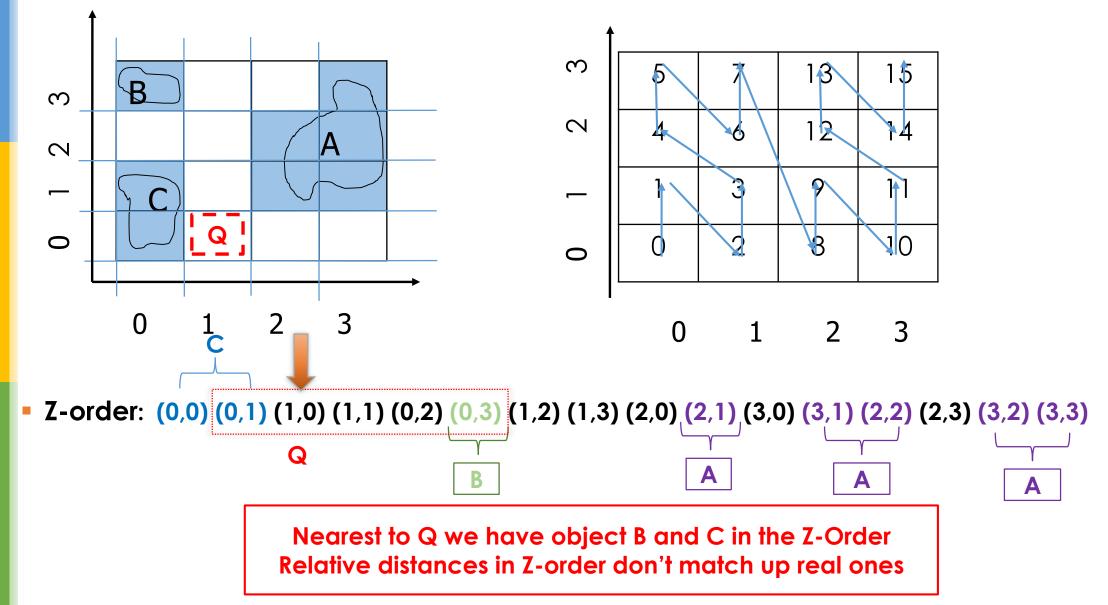
- Now take any cell (x, y) inside the query rectangle defined by LL and UU
- Using our previous argument z-value of (x,y) would be greater than z-value of LL and smaller that z-value of UR
- Basically we switch (x1,y1) and (x2, y2) with (x,y), LL, and UR to make a argument.

### Z-Order curve: K-Nearest Neighbor Query

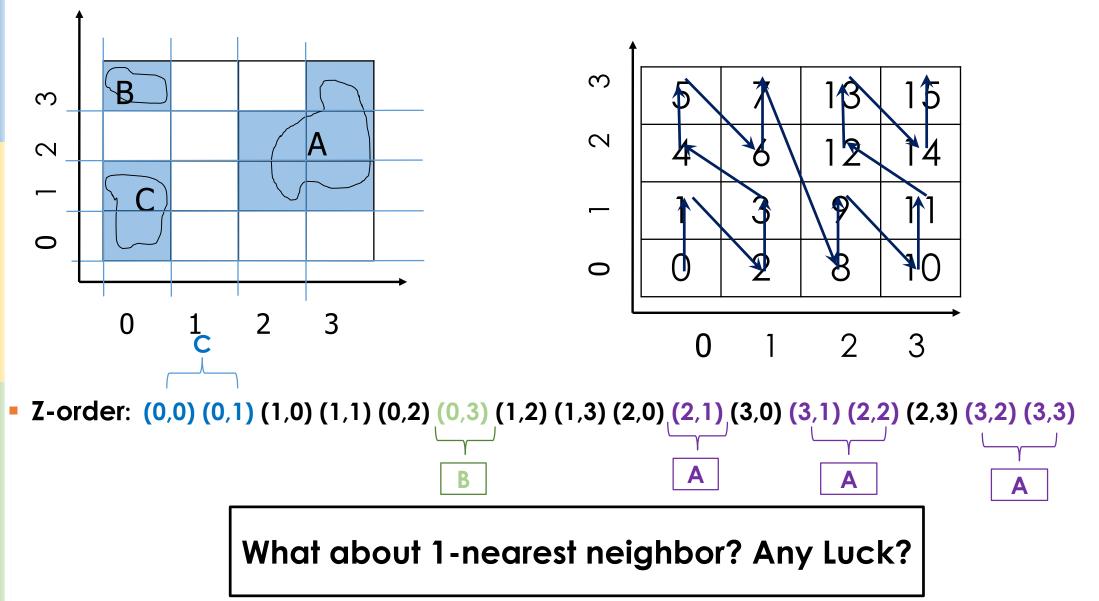


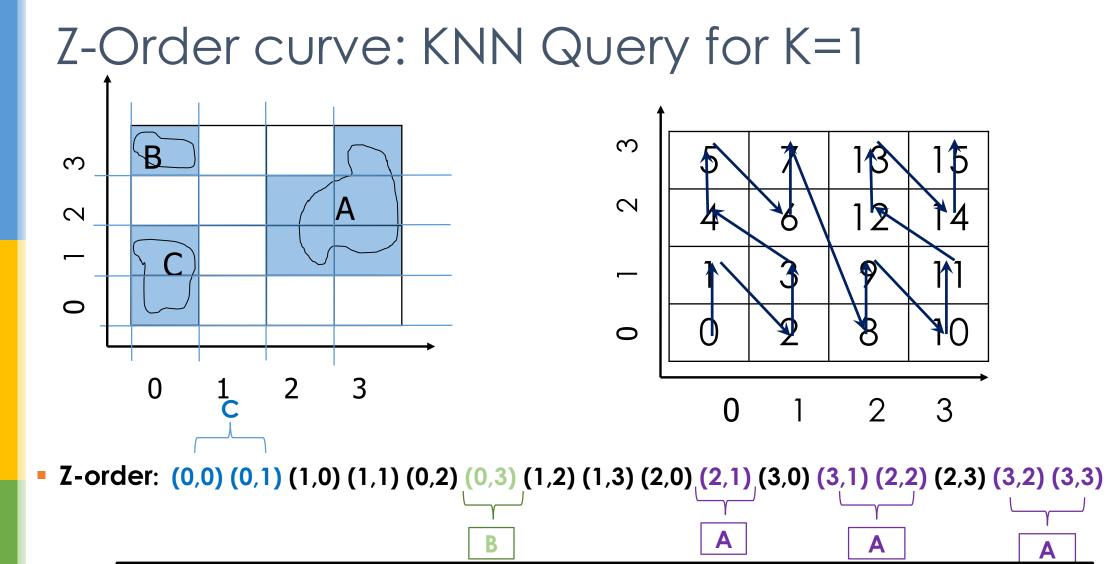
Query: What are the two closest neighbors of query point Q?

### Z-Order curve: K-Nearest Neighbor Query



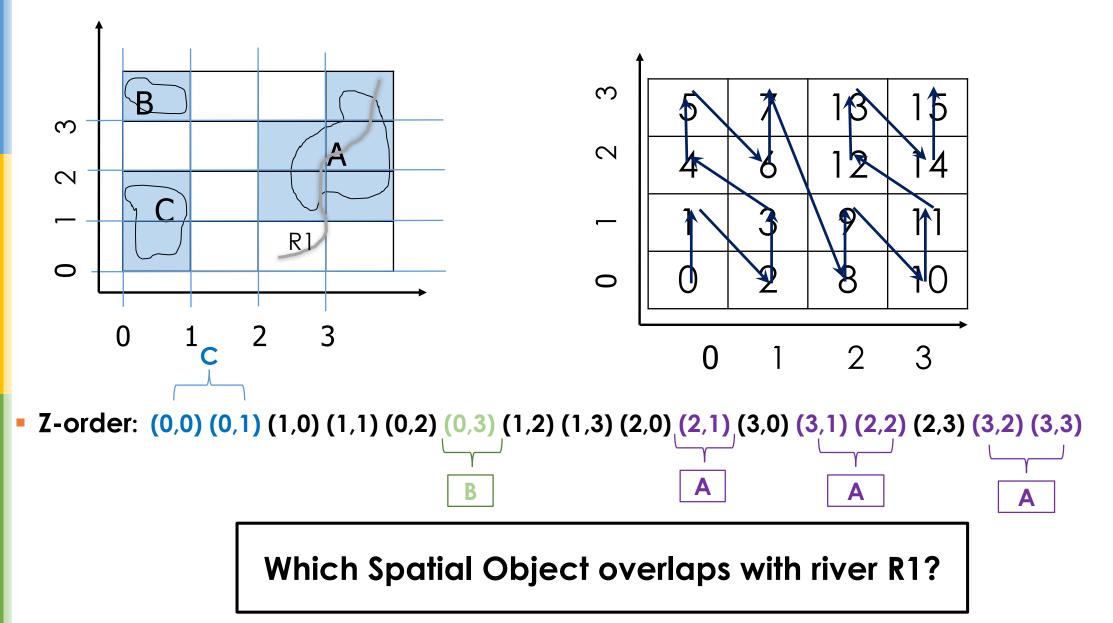
#### Z-Order curve: KNN Query for K=1



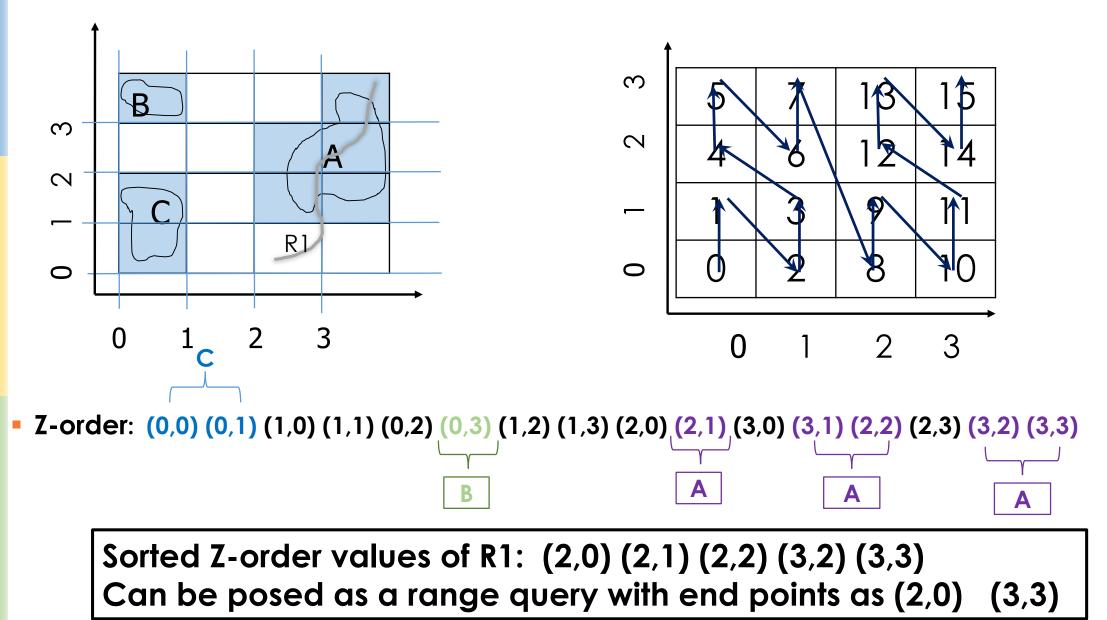


Get the NN from the z-values and issue a range query where range is a circle, query point as the center and radius is the distance to closest Z-value

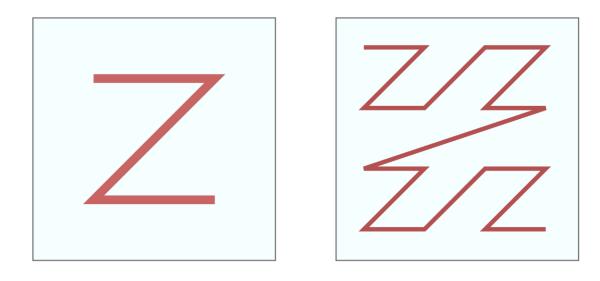
#### Z-Order curve: Algorithm for Spatial Join?

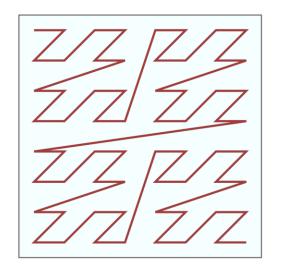


#### Z-Order curve: Algorithm for Spatial Join?



### Z-Curves in larger spaces





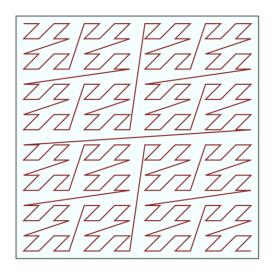
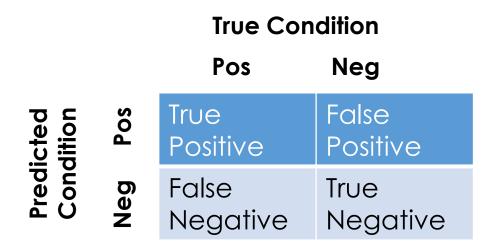


Image source: wikipedia

### Analytical Analysis of Z-Order curves

Confusion Matrix:



Precision:

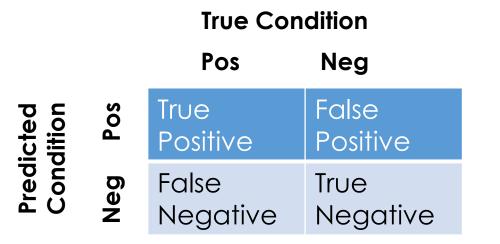
 $Precision = \frac{True Positive}{True Positive + False Positive}$ 

Recall:

 $Recall = \frac{True Positive}{True Positive + False Negative}$ 

### Analytical Analysis of Z-Order curves

#### Confusion Matrix:



Thoughts on Precision and Recall of the initial step of previous range query algorithm?

Precision:

 $Precision = \frac{True \ Positive}{True \ Positive + False \ Positive}$ 

Recall:

 $Recall = \frac{True Positive}{True Positive + False Negative}$ 

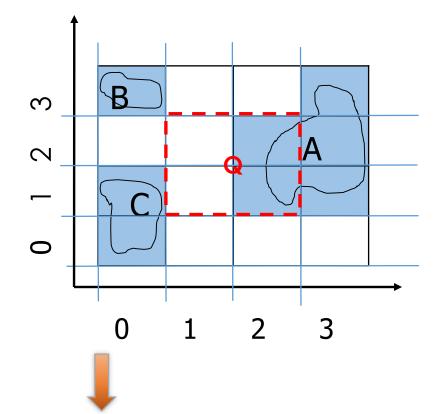
#### Analytical Analysis of Z-Order curves

Precision = True Positive True Positive + False Positive

 $Recall = \frac{True \ Positive}{True \ Positive + False \ Negative}$ 

Thoughts on Precision and Recall of the first step of the range query algorithm?

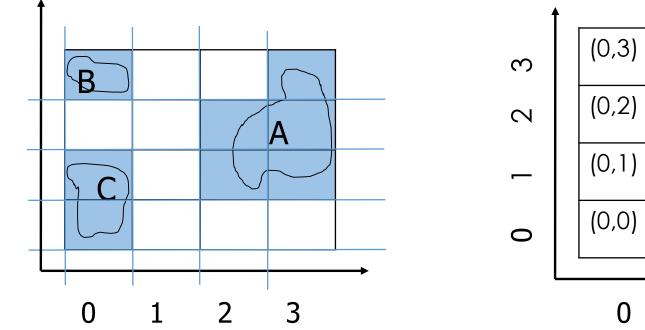
Z-order: (0,0) (0,1) (1,0) (1,1) (0,2) (0,3) (1,2) (1,3) (2,0) (2,1) (3,0) (3,1) (2,2) (2,3) (3,2) (3,3)

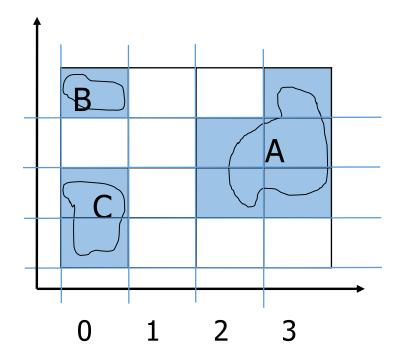


### Hilbert Curves

- **Step1:** Read in the n-bit binary representation of the x and y coordinates.
- **Step 2:** Interleave bits of the two binary numbers into one string
- **Step3:** Divide the string into from left to right into 2-bit strings
- Step4: Assign decimal values: "00" as 0; "01" as 1; "10" as 3; "11" as 2 and put into an array is the same order as the strings occurred.
- **Step5:** For each number j in the array
  - If j==0 then switch every following occurrence of 1 to 3 and vice-versa
  - If j==3 then switch every following occurrence of 0 to 2 and vice-versa
- Step6: Convert each number in the array to its binary representation (2-bit strings), concatenate from left to right and convert to decimal.

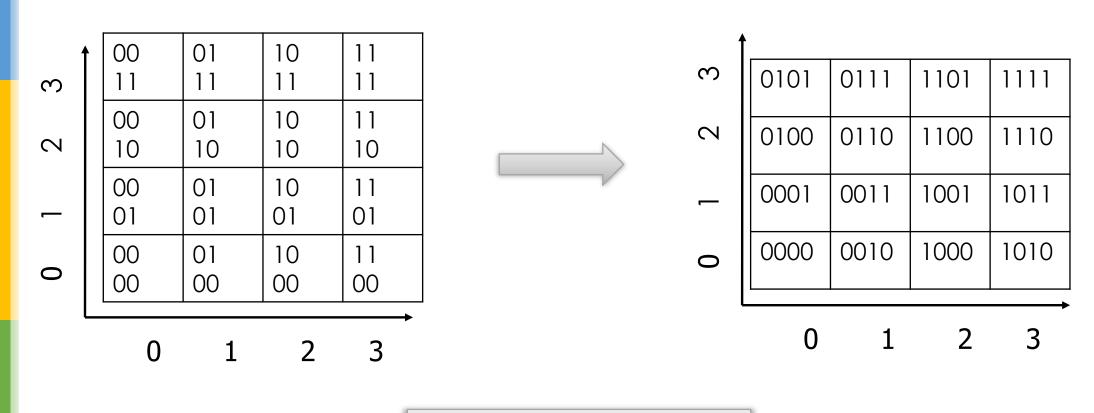
### Hilbert Curves



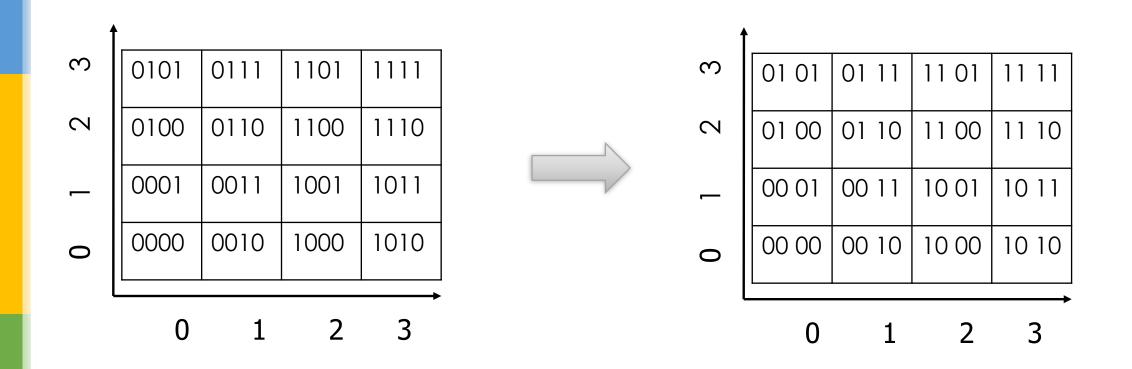


33	00	01	10 11	11
2	00	01	10	11
	10	10	10	10
1	00	01	10	11
	01	01	01	01
0	00	01	10	11
	00	00	00	00
	0	1	2	3

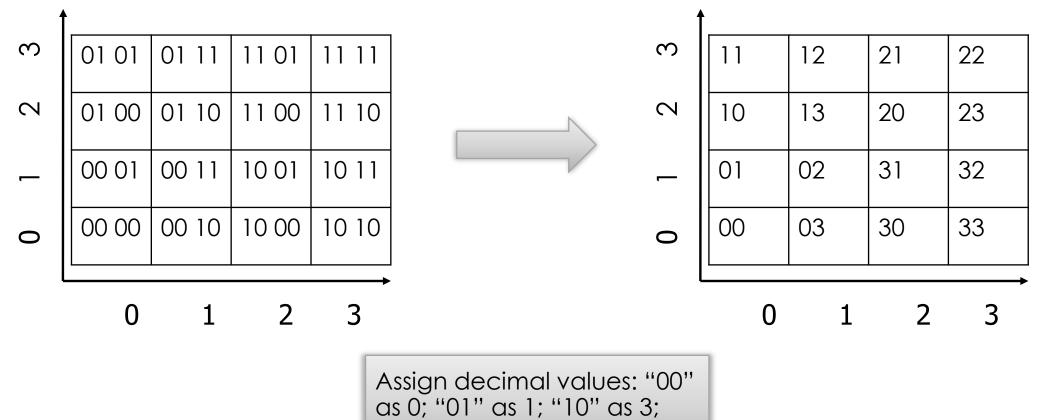
Write the X and Y coordinates in Binary Form



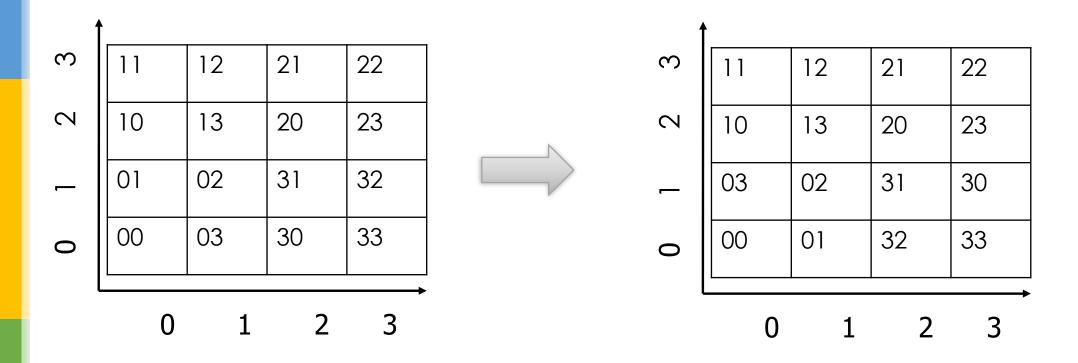
Interleave them to create one string



Divide the string into from left to right into 2-bit strings

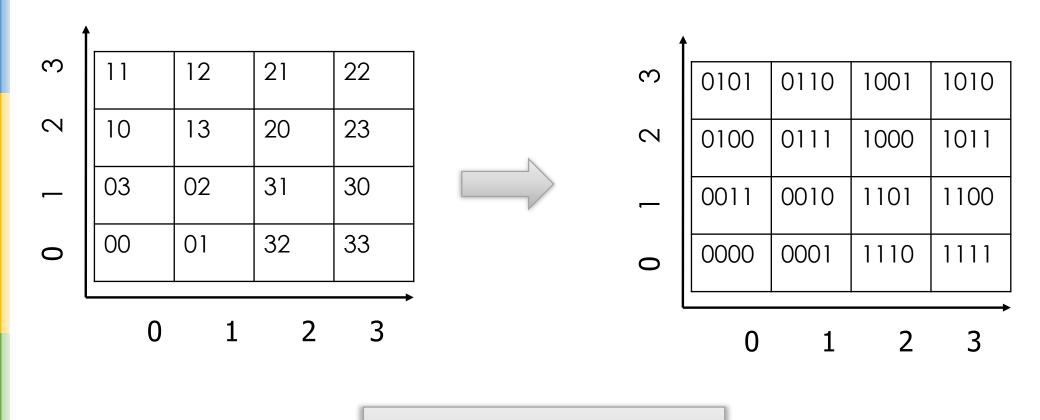


as 0; "01" as "11" as 2

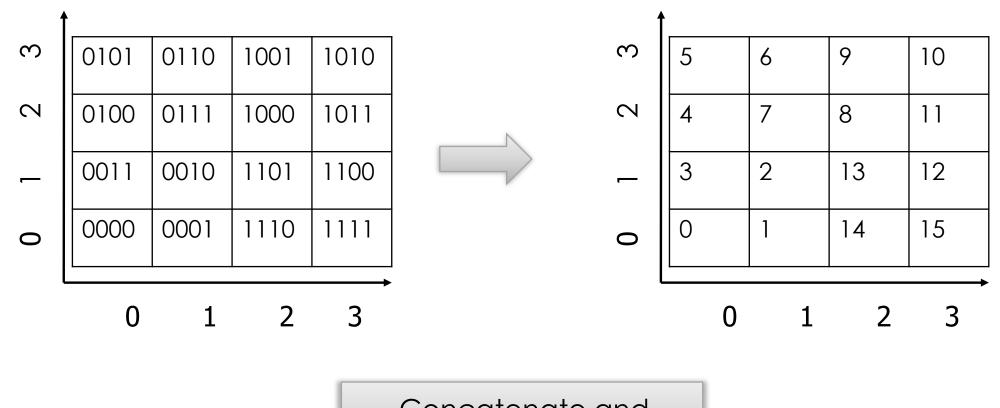


If j==0 then switch every following occurrence of 1 to 3 and vice-versa

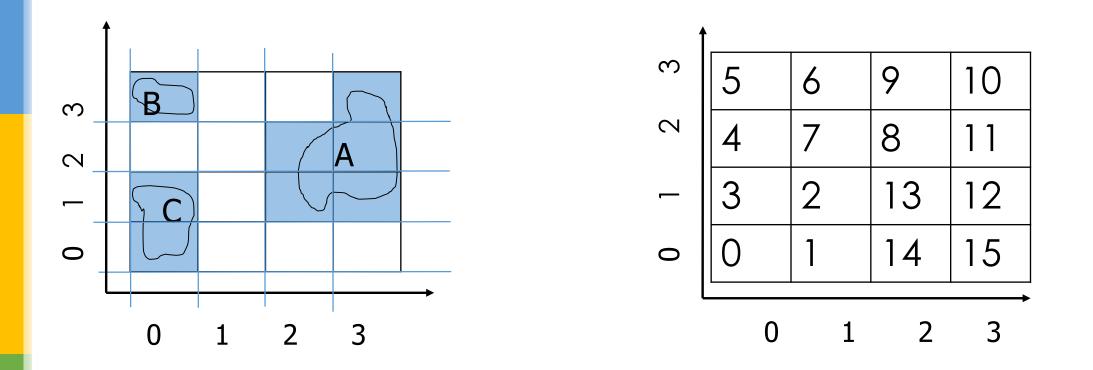
If j==3 then switch every following occurrence of 0 to 2 and vice-versa



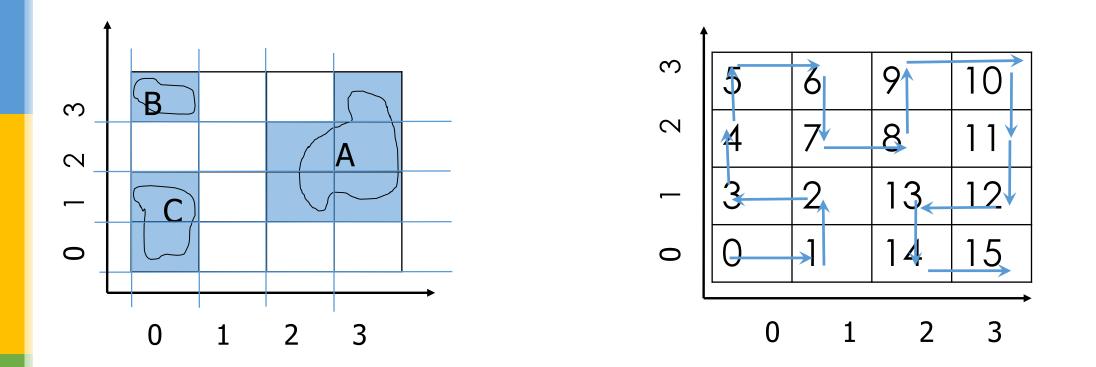
Concatenate and Convert to Binary



Concatenate and Convert to Binary

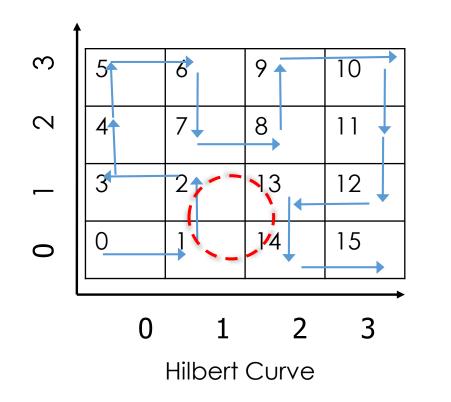


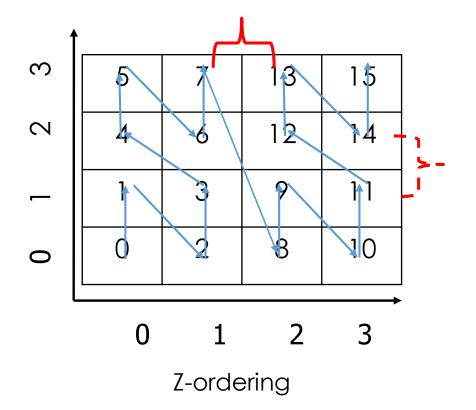
Hilbert-curve: (0,0) (1,0) (1,1) (0,1) (0,2) (0,3) (1,3) (1,2) (2,2) (2,3) (3,3) (3,2) (3,1) (2,1) (2,0) (3,0)



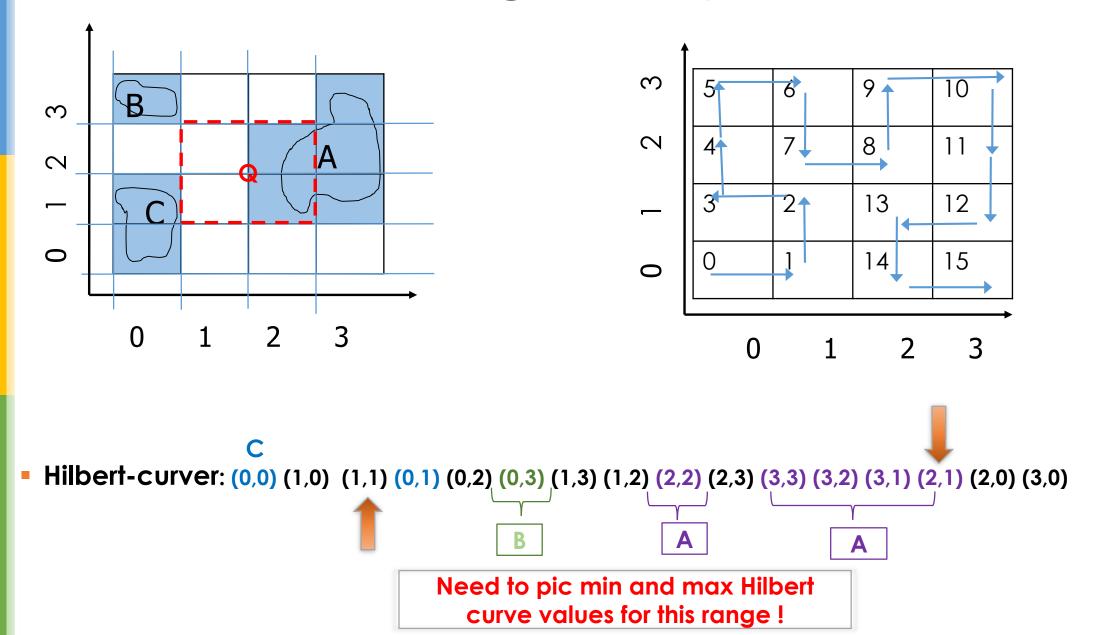
Hilbert-curve: (0,0) (1,0) (1,1) (0,1) (0,2) (0,3) (1,3) (1,2) (2,2) (2,3) (3,3) (3,2) (3,1) (2,1) (2,0) (3,0)

### Hilbert Curves Vs Z-Curves





#### Hilbert- curve: Range Query



### Hilbert Curves in larger spaces

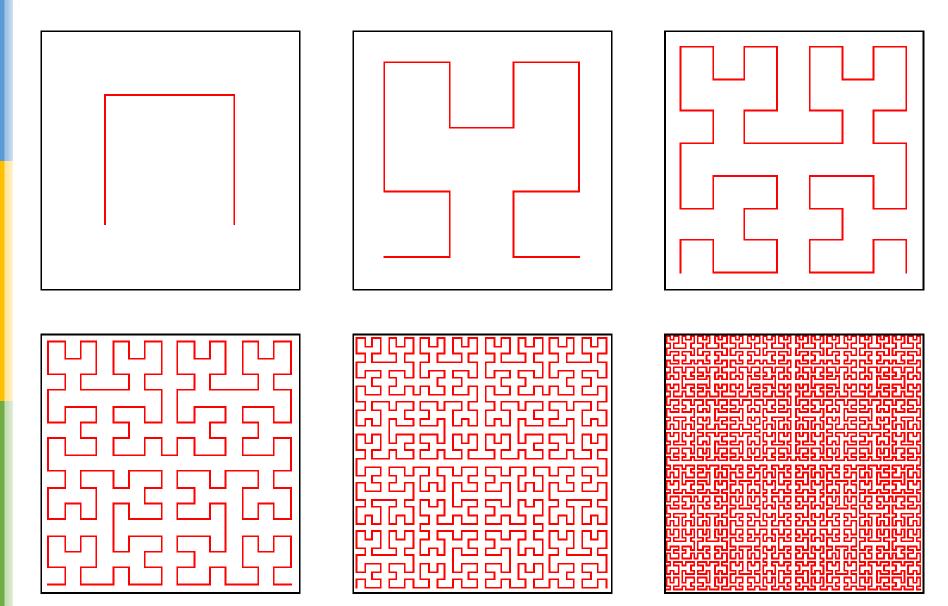
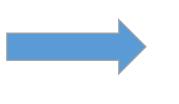


Image source and more details at: http://www.bic.m ni.mcgill.ca/~mall ar/CS-644B/hilbert.html

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0	00 00	00 10	10 00	10 10
_	00 01	00 11	10 01	10 11
2	01 00	01 10	11 00	11 10
с	01 01	01 11	11 01	11 11

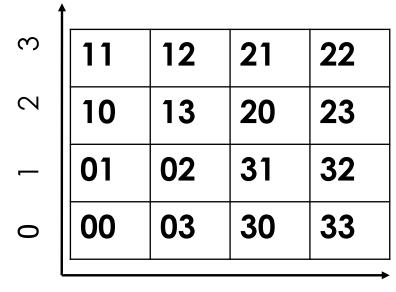


4	·			
3	11	12	21	22
2	10	13	20	23
1	01	02	31	32
0	00	03	30	33
	0	1	2	З

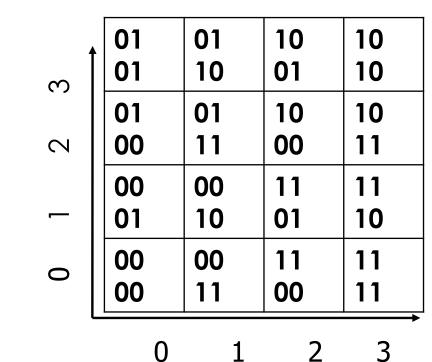
0 1 2 3

Output after Step 4

#### Say we Skip Step 5 and Jump to step 6

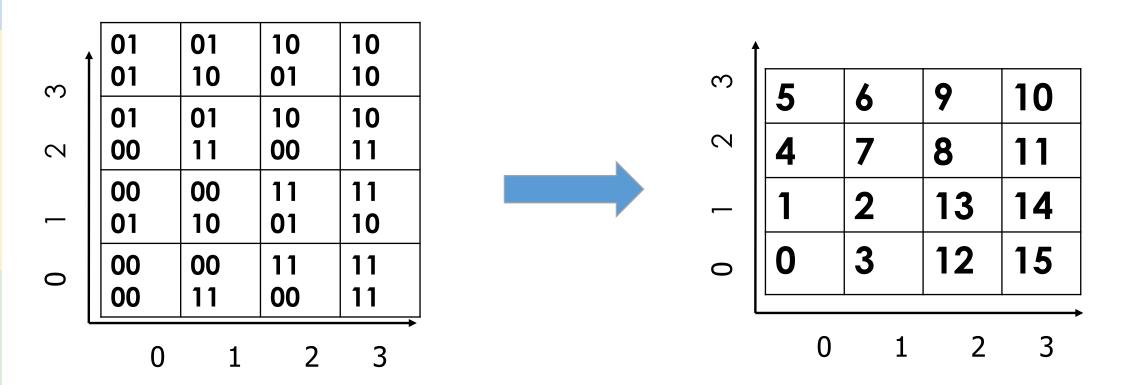


0 1 2 3



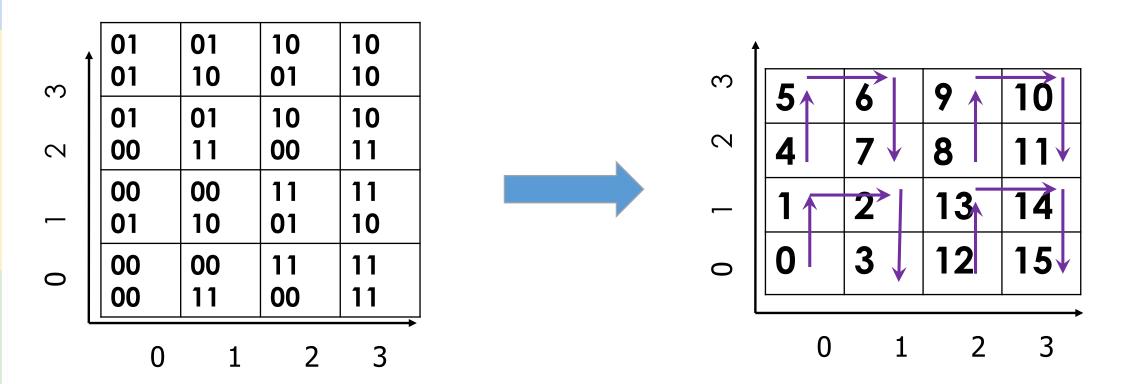
Step 6: We convert nums to binary, concatenate and then convert to decimal

Say we Skip Step 5 and Jump to step 6

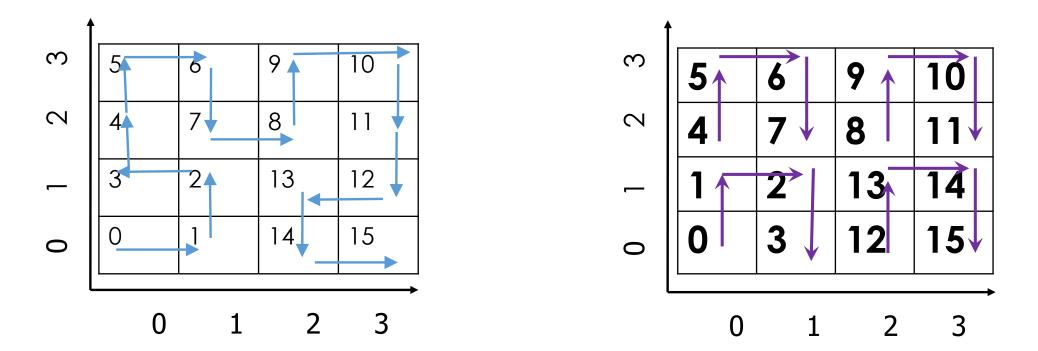


Step 6: We convert nums to binary, concatenate and then convert to decimal

Say we Skip Step 5 and Jump to step 6



Step 6: We convert nums to binary, concatenate and then convert to decimal



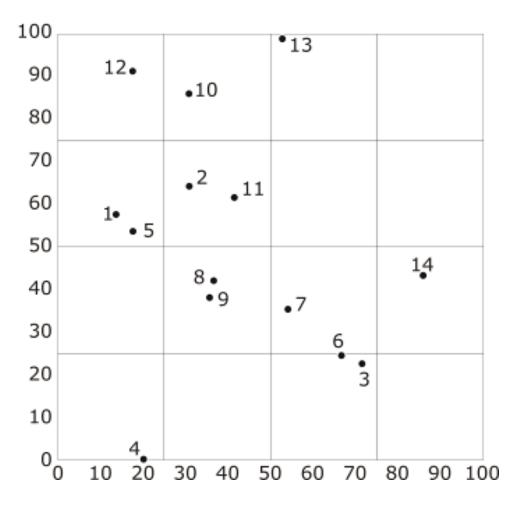
Step 5 seems to be taking care of the rotation and the reflection of the basic shape inverted cup!!!

Addressing challenges of 2-Dimenions more directly

#### Grid Files

# **Basic idea-** Divide space into cells by a grid

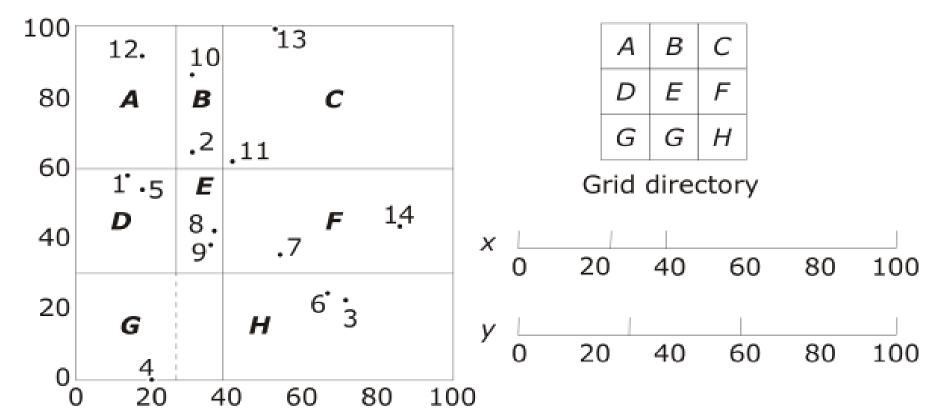
- Store data in each cell in distinct disk page
- A directory structure needed
- Efficient for find, insert, range and nearest neighbor
- May have wastage of disk storage space
- Non-uniform data distribution over space ??



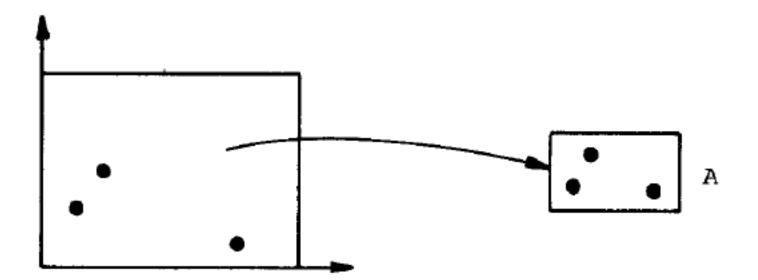
#### Grid Files

#### Refinement of basic idea into Grid Files

- Use non-uniform grids
- Linear scale store row and column boundaries
- Allow sharing of disk pages across grid cells



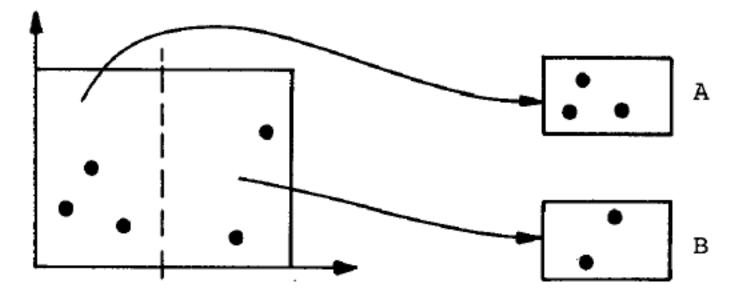
```
Grid Files (insertion example)Capacity of bucket = 3
```



J. Nievergelt and H. Hinterberger. The Grid File: An Adaptable, Symmetric Multikey File Structure. ACM Transactions on Database Systems, Vol. 9, No. 1, March 1994

#### Grid Files (insertion example)

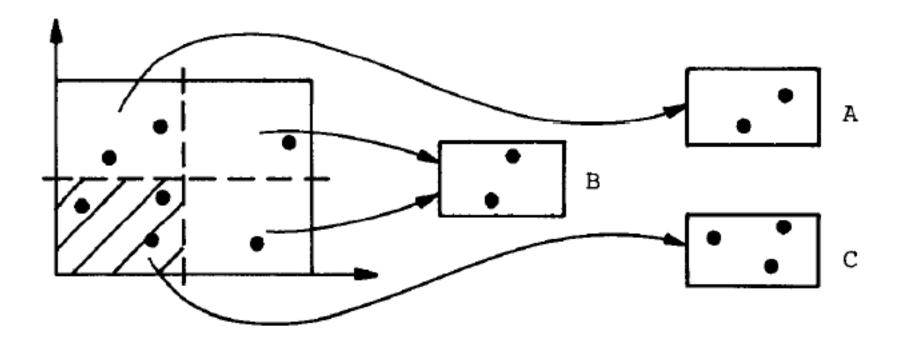
- When the bucket overflows we split it.
- A new bucket is made.
- Records that lie in one half of the space are moved to the new bucket.



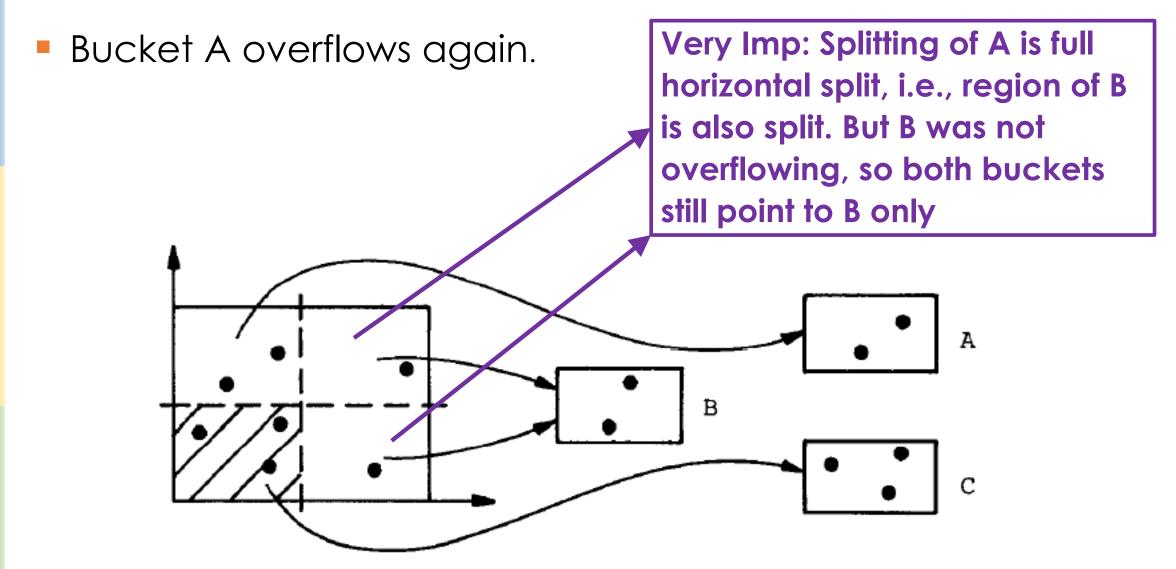
J. Nievergelt and H. Hinterberger. The Grid File: An Adaptable, Symmetric Multikey File Structure. ACM Transactions on Database Systems, Vol. 9, No. 1, March 1994

#### Grid Files (insertion example)

Bucket A overflows again.

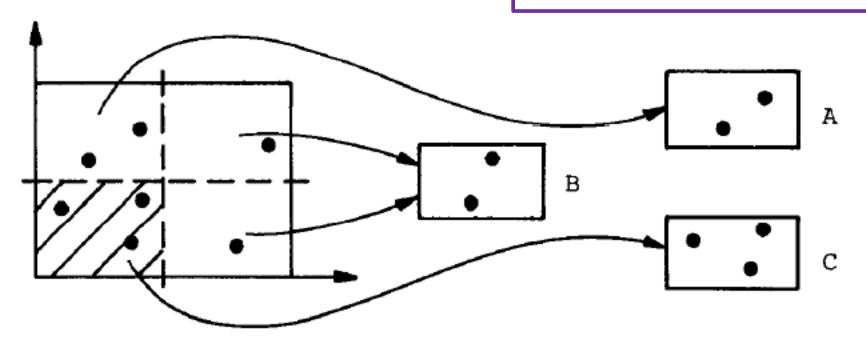


J. Nievergelt and H. Hinterberger. The Grid File: An Adaptable, Symmetric Multikey File Structure. ACM Transactions on Database Systems, Vol. 9, No. 1, March 1994



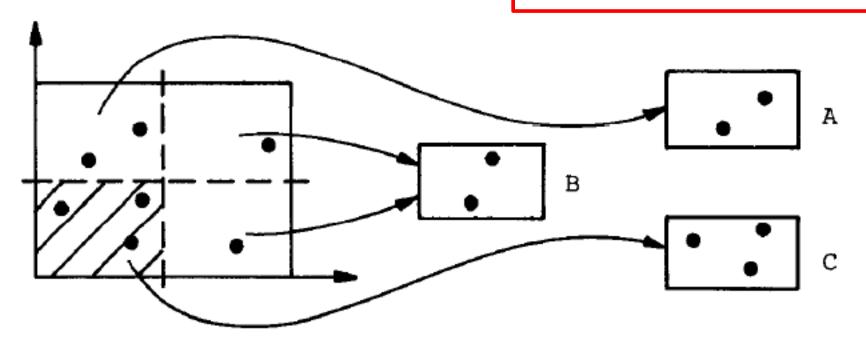
Bucket A overflows again.

In Grid files, data space which are the buckets is different from the geographic spread of the data.

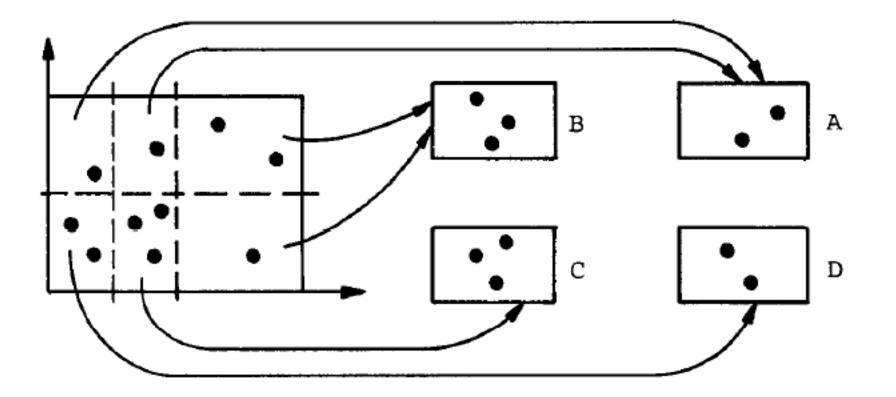


Bucket A overflows again.

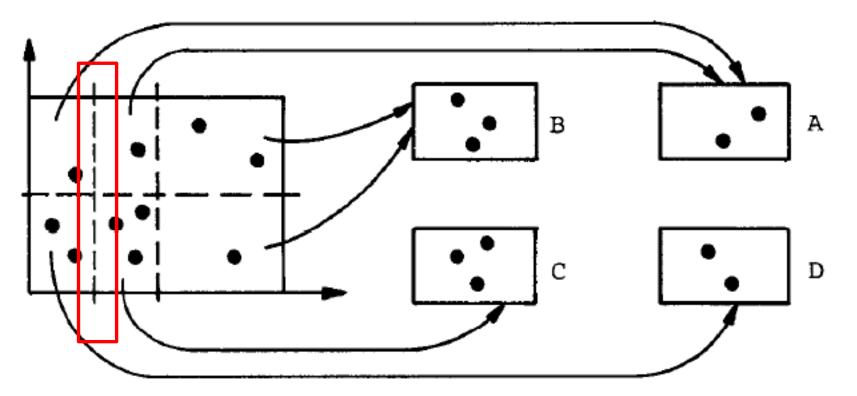
Splits in any dimension are made through and trough out. This makes the task of maintain linear scales easy

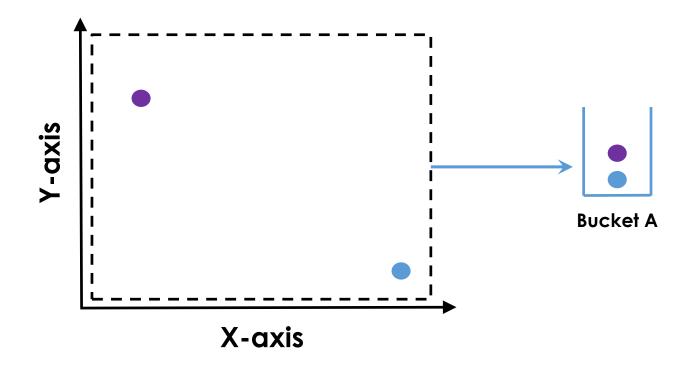


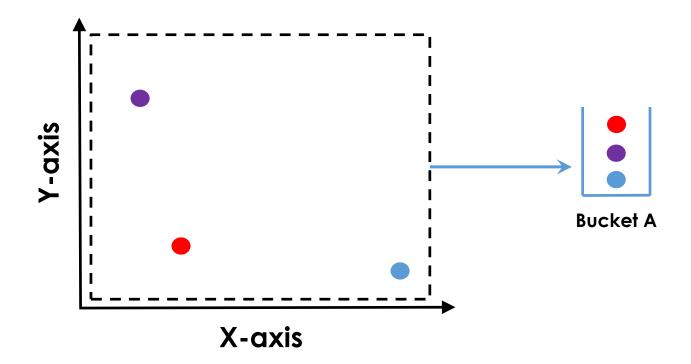
• One more split.

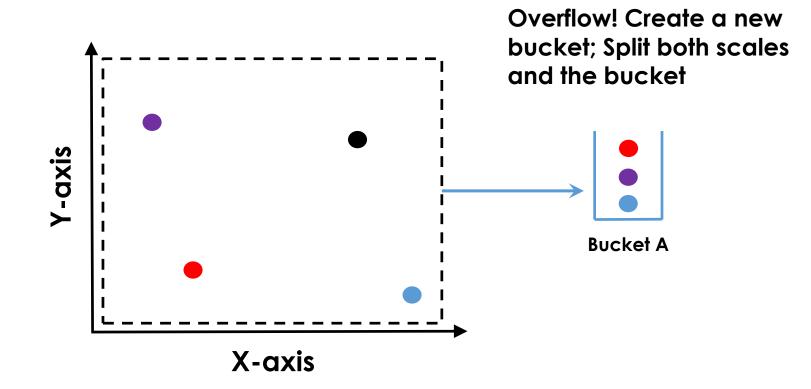


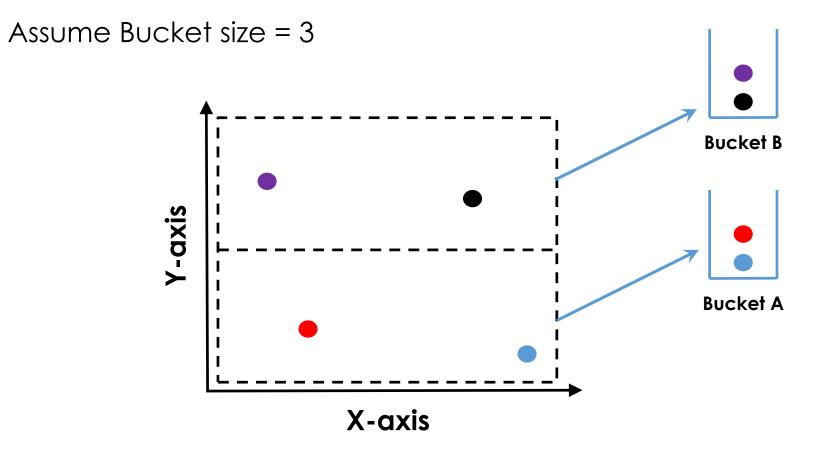
- One more split.
- Note that splits in any dimension are made through and trough.

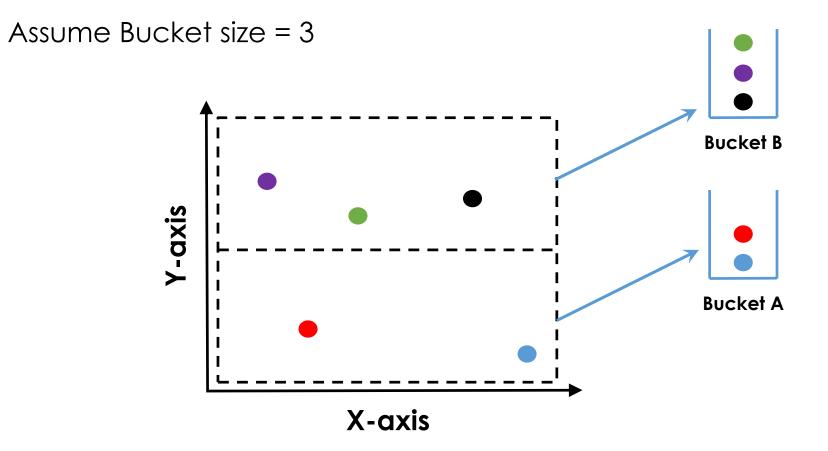




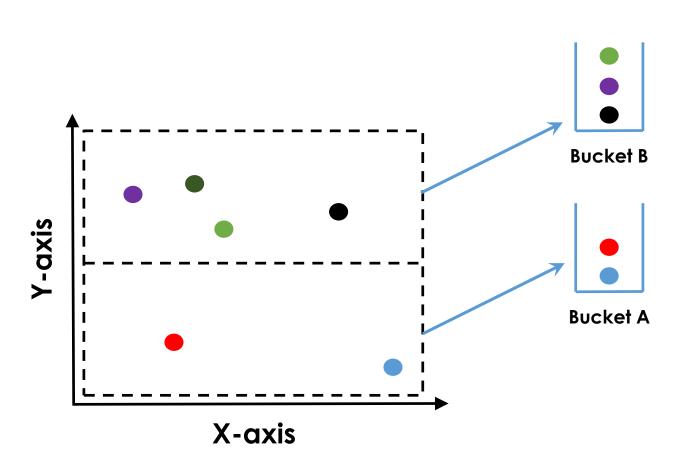




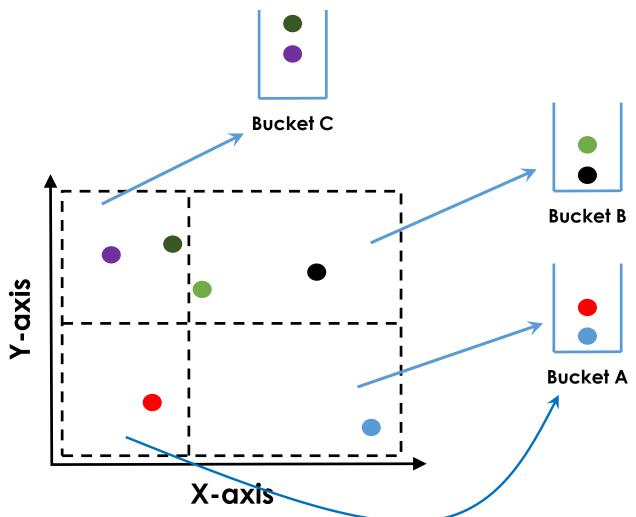


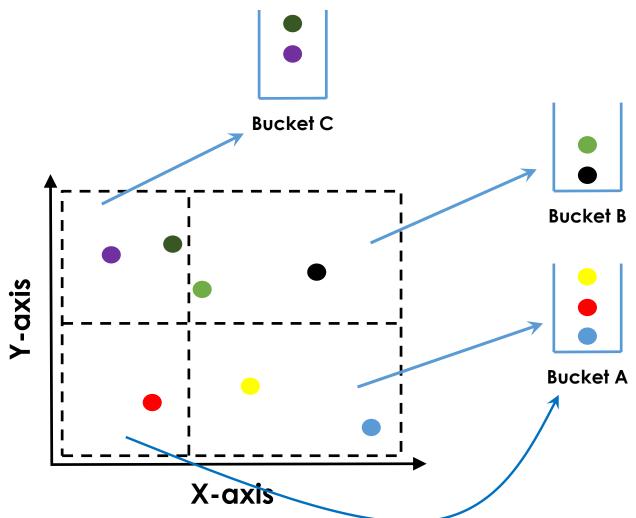


Assume Bucket size = 3

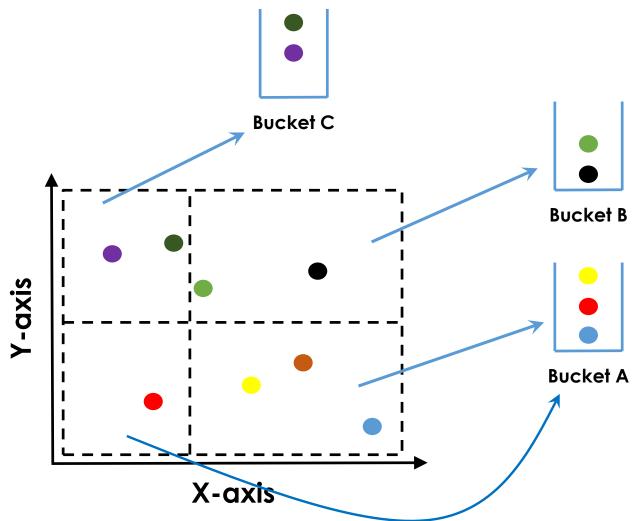


Overflow! Create a new bucket; Split both scales and the bucket.

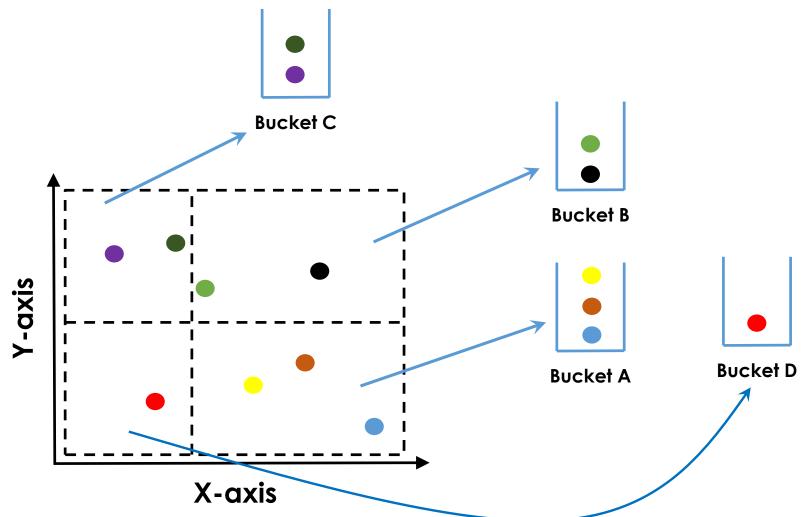


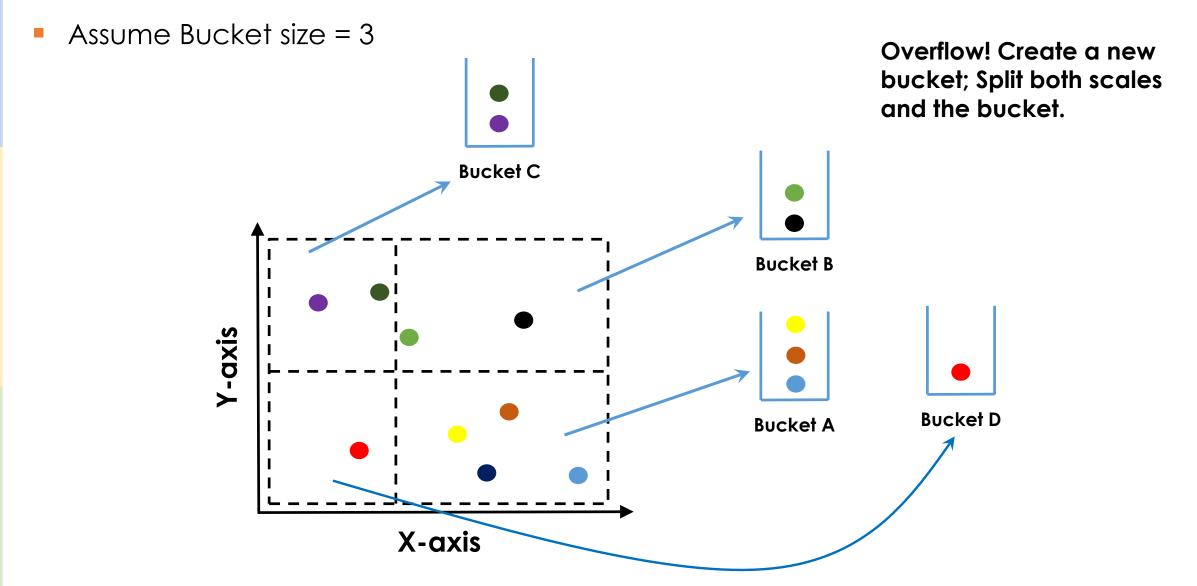


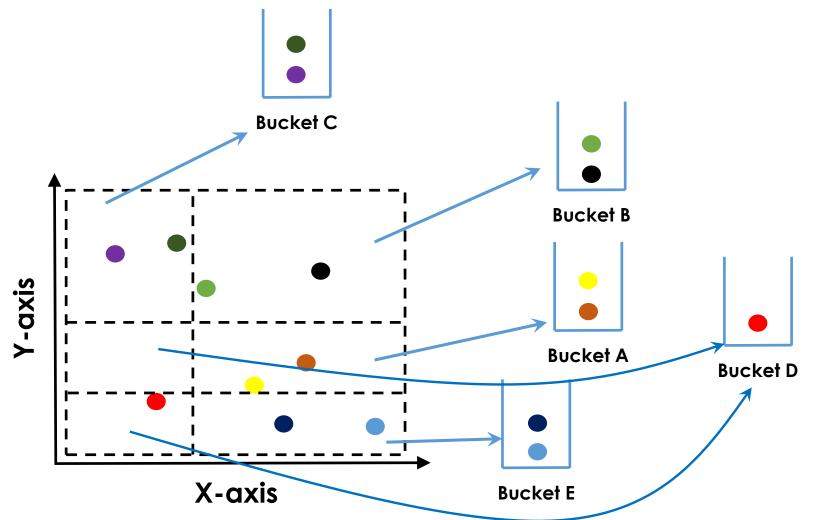
Assume Bucket size = 3



Overflow! Create a new bucket. Split bucket A.







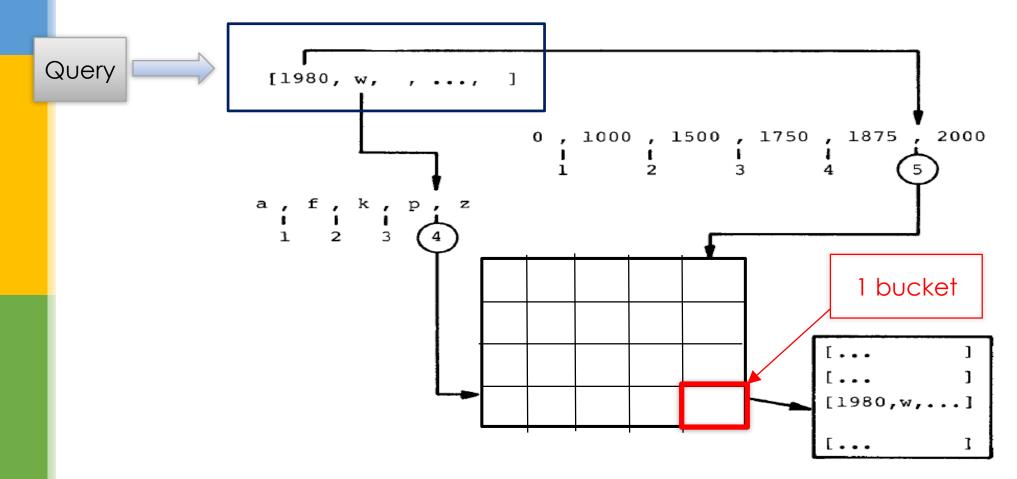
# Grid Files (Splitting Policies)

#### Splits:

- Can happen during insertion.
- Overflow of a bucket corresponding to a grid partition leads to a split.
- Can also happen if bucket containing records from several grid partition fills up.
- Splitting dimension can be changed alternatively.
- Splitting point may not always be the middle point, other algorithms are also possible.

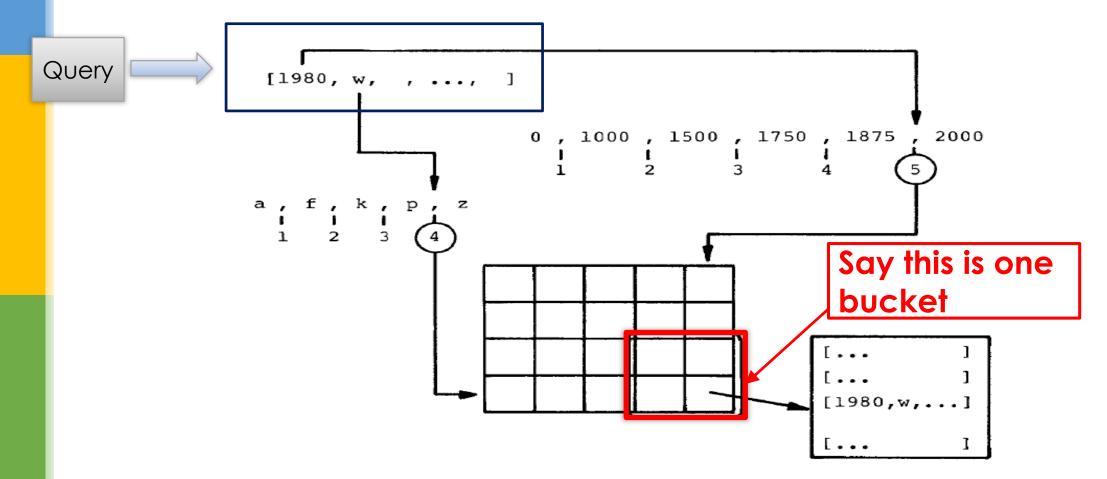
# Grid Files (Querying example)

- X-partitions (0,1000,1500,1750,1875,2000)
- Y-partitions (a, f, k, p, z).



# Grid Files (Querying example)

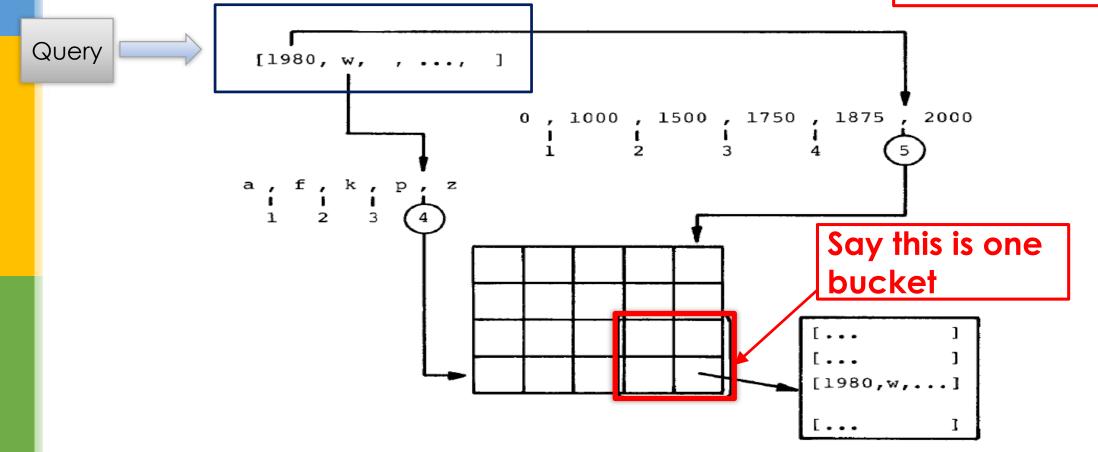
- X-partitions (0,1000,1500,1750,1875,2000)
- Y-partitions (a, f, k, p, z).



# Grid Files (Querying example)

- X-partitions (0,1000,1500,1750,1875,2000)
- Y-partitions (a, f, k, p, z).

Thoughts on Precision and Recall of the initial step of this algorithm?



# Grid Files (Merging Policies)

#### Merging:

- Happens when data is being deleted.
- Buckets may be merged in case of underflow.
- Multiple policies can be developed for merging.
- Details beyond the scope of this course.
- Interested readers can refer the paper for details.