## Introduction to Spatial Computing CSE 555

Spatial Indexing Techniques for Secondary Memory

## Scenario for Designing Spatial Indexes

- Goal: Store spatial objects A,B and
 C in storage system such that following queries can be executed efficiently.
- Point Queries:
- Range Queries:
- Nearest Neighbor Queries
- Spatial Joins:


## Scenario for Designing Spatial Indexes

- Goal: Store spatial objects A,B and C
 in storage system such that following queries can be executed efficiently.
- Point Queries:
- Given an object search if it exists in the database or not
- Example: Return the spatial object located at $(3,2)$
- Range Queries:
- Nearest Neighbor Queries
- Spatial Joins:


## Scenario for Designing Spatial Indexes

- Goal: Store spatial objects A,B and C
 in storage system such that following queries can be executed efficiently.
- Point Queries:
- Range Queries:
- Return the objects which lie within the defined range of $x$ and $y$
- Example: return objects which lie in the rectangle defined by $0<x<2$ and $0<y<2$
- Nearest Neighbor Queries
- Spatial Joins:


## Scenario for Designing Spatial Indexes

- Goal: Store spatial objects A,B and C
 in storage system such that following queries can be executed efficiently.
- Point Queries:
- Range Queries:
- Nearest Neighbor Queries
- Find the nearest spatial object (or $k$ nearest spatial objects) of the point $(2,1)$
- Spatial Joins:


## Scenario for Designing Spatial Indexes

- Goal: Store spatial objects A,B and C
 in storage system such that following queries can be executed efficiently.
- Point Queries:
- Range Queries:
- Nearest Neighbor Queries
- Spatial Joins:
- Find the spatial objects which intersect the object R1


## Scenario for Designing Spatial Indexes



- Had these objects been a 1-dimensional in nature, e.g., real numbers, strings etc.
- A simple B+ tree would be constructed over these.
- Can easily get $\mathrm{O}(\log \mathrm{n})$ complexity for all the queries (except the join query) mentioned in the previous slides.


## Scenario for Designing Spatial Indexes



- How to get ordering in 2Dimensions?
- Once we get ordering we can try B+ tree again for spatial objects.


## Towards Getting an Order Basics

- Approximate objects with cells.

- Helps in getting a continuous space to work with easer to handle.
- Would have to map back whenever necessary (for the queries and results).


## Towards Getting an Order



First Attempt

- Order on Y then X: $(0,0)(1,0)(2,0)$ $(3,0)(0,1)(1,1)(2,1)(3,1)(0,2)(1,2)$ $(2,2)(3,2)(0,3)(1,3)(2,3)(3,3)$


## Towards Getting an Order



## First Attempt

- Order on $Y$ then $X:(0,0)(1,0)(2,0)(3,0)$ $(0,1)(1,1)(2,1)(3,1)(0,2)(1,2)(2,2)$ $(3,2)(0,3)(1,3)(2,3)(3,3)$
- Insert tuples <(0,0), C>; <(0,1),C>; $\langle(2,1), A\rangle$; $\langle(3,1), A\rangle$; $\langle(2,2), A\rangle$; $<(3,2), A>$; $\langle(0,3), B>$; $<(3,3), A>$; in a $B+$ tree.
- These would be order of leaves in the B+ tree


## Towards Getting an Order



First Attempt ( Y then X )

- Insert tuples $\langle(0,0), C\rangle ;<(0,1), C>;$ $\langle(2,1), A\rangle$; $\langle(3,1), A\rangle ;<(2,2), A\rangle$; $\langle(3,2), A>$; $\langle(0,3), B\rangle$; <(3,3),A>; in a $B+$ tree.
- Range Query: Retrieve the objects whose $2=<x=<3$ and $2=<y=<3$


## Towards Getting an Order

- First Attempt (Y then X)

- Insert tuples $\langle(0,0), C\rangle$; $(0,1), C\rangle$; $\langle(2,1), A\rangle$; $\langle(3,1), A\rangle$; $\langle(2,2), A\rangle$; $\langle(3,2), A\rangle$; $\langle(0,3), B\rangle$; $\langle(3,3), A\rangle$; in a $B+$ tree.
- Range Query: Retrieve the objects whose $2=<x=<3$ and $2=<y=<3$

Not really in the
range but still got
in

## Towards Getting an Order



Second Attempt ( X then Y )

## Order on $X$ then $Y$ :

$(0,0)(0,1)(0,2)(0,3)(1,0)(1,1)(1,2)$
$(1,3)(2,0)(2,1)(2,2)(2,3)(3,0)(3,1)$
$(3,2)(3,3)$

## Towards Getting an Order



Second Attempt ( $X$ then $Y$ ) Order on $X$ then $Y$ :
$(0,0)(0,1)(0,2)(0,3)(1,0)(1,1)$
$(1,2)(1,3)(2,0)(2,1)(2,2)(2,3)$
$(3,0)(3,1)(3,2)(3,3)$

Range Query $2<x<3 \& 2<y<3$ : Little better this time

## How about in this scenario?



Range Query: Retrieve all objects in this range $1=<x=<2 \& y=3$ ?

## Towards Getting an Order

Neighboring Cells but far apart in the ordering


- Problem with these orderings: Cells which are close to each other might get spread out and occupy places quite far from each other.
- Need a ordering which can preserve spatial locality in both $x$ and $y$ directions as much as possible!
- Cannot get 100\%


## Z-Order curve



| $(0,3)$ | $(1,3)$ | $(2,3)$ | $(3,3)$ |
| :---: | :---: | :---: | :---: |
| $(0,2)$ | $(1,2)$ | $(2,2)$ | $(3,2)$ |
| $(0,1)$ | $(1,1)$ | $(2,1)$ | $(3,1)$ |
| $(0,0)$ | $(1,0)$ | $(2,0)$ | $(3,0)$ |
| 0 | 1 | 2 | 3 |

## Z-Order curve




Write the $X$ and $Y$ coordinates in Binary Form

## Z-Order curve



| 0101 | 0111 | 1101 | 1111 |
| :---: | :---: | :---: | :---: |
| 0100 | 0110 | 1100 | 1110 |
| 0001 | 0011 | 1001 | 1011 |
| 0000 | 0010 | 1000 | 1010 |
| 0 | 1 | 2 | 3 |

Interleave them to create one string

## Z-Order curve




Convert the bit strings to its corresponding decimal

## Z-Order curve




This is the order of cells from this process

## Z-Order curve




This is the order of cells from this process

## Z-Order curve




Visually its looks like we have Zs on our map.
Hence the name Z-order curve!!

## Z-Order curve




Ordering: X followed by Y

Fewer neighboring cells are far in the ordering


## Z-Order curve: Range Query




Z-order: $(0,0)(0,1)(1,0)(1,1)(0,2)$
$\underbrace{(0,3)}_{\square}(1,2)(1,3)(2,0)(\underbrace{(2,1)}_{\square}(3,0)(\underbrace{(3,1)}_{\square}(2,2)(2,3)(\underbrace{(3,2)}_{\square}(3,3)$

## Z-Order curve: Range Query




Z-order: $(0,0)(0,1)(1,0)(1,1)(0,2)(0,3)(1,2)(1,3)(2,0)(2,1)(3,0)(3,1)(2,2)(2,3)(3,2)(3,3)$


## Z-Order curve: Range Query




Z-order: $(0,0)(0,1)(1,0)(1,1)(0,2)(0,3)(1,2)(1,3)(2,0)(2,1)(3,0)(3,1)(2,2)(2,3)(3,2)(3,3)$


## Correctness of Range Query on Z-Order curves

- Consider again our previous example:


- Z-order: $(0,0)(0,1)(1,0)(1,1)(0,2)(0,3)(1,2)(1,3)(2,0)(2,1)(3,0)(3,1)(2,2)(2,3)(3,2)(3,3)$
- Retrieved all records within this range and cross checked the result.


## Correctness of Range Query on Z-Order curves

- Consider again our previous example:


Right answer would certainly be in the result. But it may contain some other information also which need to cleaned

- Z-order: $(0,0)(0,1)(1,0)(1,1)(0,2)(0,0)(1,2)(1,0)(2,0)(2,1)(0,0)(0,1)(2,2)(2,3)(3,2)(3,3)$
- Retrieved all records within this range and cross checked the result.


## Correctness of Range Query on Z-Order curves

## Proof Sketch:

- Z-order: $(0,0)(0,1)(1,0)(1,1)(0,2)(0,3)(1,2)(1,3)(2,0)(2,1)(3,0)(3,1)$ $(2,2)(2,3)(3,2)(3,3)$
- Retrieved all records within this range and cross checked the result.
- For this approach to be correct we need to prove that all the cells which are in the query rectangle of $(1,1)$ and $(2,2)$ are between 4 and 9 .


## Correctness of Range Query on Z-Order curves

## Proof Sketch:

- Without loss of generalization let:
- LL = (xmin, ymin) is the lower left of the query rectangle
- UR = (xmax, ymax) is the upper right of the query rectangle.
- Then we need to prove that all the cells with ( $x$ min $<\mathbf{x}<$ xmax) and ( ymin < y < ymax) will have their $\mathbf{Z}$-values between z-values of LL and UR.


## Correctness of Range Query on Z-Order curves

## Proof Sketch:

- Take two cell coordinates numbers: $(x 1, y 1)$ and ( $x 2, y 2$ )
- Case I: x2 > x1 and y1 = y2
" If $x 2$ is greater than $x 1$ that it will have " 1 " in at least one higher position in binary form
" Which means it will get " 1 " in at least one higher position in its z-value.
- Implies that it will have a higher z-value.


## Correctness of Range Query on Z-Order curves

## Proof Sketch:

- Case II: y2 > y1 and $x 1=x 2$
- If y 2 is greater than y 1 that it will have " 1 " in at least one higher position in binary form
- Which means it will get " 1 " in at least one higher position in its $z$-value
- Implies it will have a higher z-value.


## Correctness of Range Query on Z-Order curves

## Proof Sketch:

- Case III: x2 > x1 and y2 > y1
- Similar argument of getting " 1 " in at least one higher position in its z-value
- Implies it will have a higher z-value


## Correctness of Range Query on Z-Order curves

## Proof Sketch:

- Now take any cell ( $x, y$ ) inside the query rectangle defined by LL and UU
- Using our previous argument z-value of ( $x, y$ ) would be greater than z-value of LL and smaller that z-value of UR
- Basically we switch ( $\mathrm{x} 1, \mathrm{y} 1$ ) and ( $\mathrm{x} 2, \mathrm{y} 2$ ) with ( $\mathrm{x}, \mathrm{y}$ ), LL, and UR to make a argument.


## Z-Order curve: K-Nearest Neighbor Query





> Query: What are the two closest neighbors of query point Q?

## Z-Order curve: K-Nearest Neighbor Query


$\begin{array}{lll}0 & 1 & 2\end{array}$
3


Z-order: $(0,0)(0,1)(1,0)(1,1)(0,2)(0,3)(1,2)(1,3)(2,0) \underbrace{(2,1)}_{\mathrm{Q}}(3,0) \underbrace{(3,1)}_{\square}(2,2)(2,3)(\underbrace{(3,2)}_{\mathrm{B}}(3,3)$ Relative distances in Z-order don't match up real ones

## Z-Order curve: KNN Query for K=1 <br>  <br> 



## What about 1-nearest neighbor? Any Luck?

Z-Order curve: KNN Query for K=1



Get the NN from the z-values and issue a range query where range is a circle, query point as the center and radius is the distance to closest Z-value

## Z-Order curve: Algorithm for Spatial Join?





## Which Spatial Object overlaps with river R1?

## Z-Order curve: Algorithm for Spatial Join?



" Z-order: $(0,0)(0,1)(1,0)(1,1)(0,2) \underbrace{(0,3)}_{\square}(1,2)(1,3)(2,0) \underbrace{(2,1)}_{\square}(3,0)(\underbrace{(3,1)(2,2)}_{\mathrm{B}}(2,3)(\underbrace{(3,2)(3,3)}_{\mid}$
Sorted Z-order values of R1: $(2,0)(2,1)(2,2)(3,2)(3,3)$ Can be posed as a range query with end points as $(2,0) \quad(3,3)$

## Z-Curves in larger spaces



| $\begin{aligned} & Z / 2 \\ & 3 / z \\ & Z / z \end{aligned}$ | $\sqrt{2 \pi} \sqrt{2 \pi}$ |
| :---: | :---: |
| UB | WZ 27 |
| $\pi / \pi /$ | $\sqrt{1} \pi /$ |
| \% | R/Z 27 |
|  |  |
| % | $\sqrt{7} V^{2}$ |
|  |  |
|  | 25 |
|  | $\checkmark 7$ |

## Analytical Analysis of Z-Order curves

- Confusion Matrix:

True Condition

|  | ô | Pos | Neg |
| :---: | :---: | :---: | :---: |
|  |  | True Positive | False Positive |
|  | $\begin{aligned} & \text { ס } \\ & \text { (1) } \end{aligned}$ | False Negative | True Negative |

- Precision:

$$
\text { Precision }=\frac{\text { True Positive }}{\text { True Positive }+ \text { False Positive }}
$$

- Recall:

$$
\text { Recall }=\frac{\text { True Positive }}{\text { True Positive }+ \text { False Negative }}
$$

## Analytical Analysis of Z-Order curves

- Confusion Matrix:

|  |  | True Condition |  |
| :---: | :---: | :---: | :---: |
|  |  | Pos | Neg |
|  | ŏ | True Positive | False Positive |
|  | $\begin{aligned} & \text { ס } \\ & \text { ( } \end{aligned}$ | False Negative | True Negative |

> Thoughts on Precision and Recall of the initial step of previous range query algorithm?

- Precision:

$$
\text { Precision }=\frac{\text { True Positive }}{\text { True Positive }+ \text { False Positive }}
$$

- Recall:

$$
\text { Recall }=\frac{\text { True Positive }}{\text { True Positive }+ \text { False Negative }}
$$

## Analytical Analysis of Z-Order curves

$$
\text { Precision }=\frac{\text { True Positive }}{\text { True Positive }+ \text { False Positive }}
$$

$$
\text { Recall }=\frac{\text { True Positive }}{\text { True Positive }+ \text { False Negative }}
$$

> Thoughts on Precision and Recall of the first step of the range query algorithm?


- Z-order: $(0,0)(0,1)(1,0)(1,1)(0,2)(0,3)(1,2)(1,3)(2,0)(2,1)(3,0)(3,1)(2,2)(2,3)(3,2)(3,3)$


## Hilbert Curves

- Step 1: Read in the n-bit binary representation of the $x$ and $y$ coordinates.
- Step 2: Interleave bits of the two binary numbers into one string
- Step3: Divide the string into from left to right into 2-bit strings
- Step4: Assign decimal values: "00" as 0; "01" as 1; " 10 " as 3; " 11 " as 2 and put into an array is the same order as the strings occurred.
- Step5: For each number j in the array
- If $j==0$ then switch every following occurrence of 1 to 3 and vice-versa
- If $\mathrm{j}==3$ then switch every following occurrence of 0 to 2 and vice-versa
- Step6: Convert each number in the array to its binary representation (2-bit strings), concatenate from left to right and convert to decimal.


## Hilbert Curves



| m | $(0,3)$ | $(1,3)$ | $(2,3)$ | $(3,3)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\sim$ | $(0,2)$ | $(1,2)$ | $(2,2)$ | $(3,2)$ |
| - | $(0,1)$ | (1,1) | $(2,1)$ | $(3,1)$ |
| $\bigcirc$ | $(0,0)$ | $(1,0)$ | $(2,0)$ | $(3,0)$ |
|  | 0 | 1 | 2 | 3 |

## Hilbert Curves (Step 1)



| m | $\begin{aligned} & \hline 00 \\ & 11 \end{aligned}$ | $\begin{aligned} & 01 \\ & 11 \end{aligned}$ | $\begin{aligned} & 10 \\ & 11 \end{aligned}$ | $\begin{aligned} & 11 \\ & 11 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| N | 00 | 01 | 10 | 11 |
|  | 10 | 10 | 10 | 10 |
| - | 00 | 01 | $\begin{aligned} & 10 \\ & 01 \end{aligned}$ | $\begin{aligned} & 11 \\ & 01 \end{aligned}$ |
| $\bigcirc$ | 00 00 | 01 00 | $\begin{aligned} & 10 \\ & 00 \end{aligned}$ | $\begin{aligned} & 11 \\ & 00 \end{aligned}$ |

Write the $X$ and $Y$
coordinates in Binary Form

## Hilbert Curves (Step 2)

| m | $\begin{array}{\|l\|} \hline 00 \\ 11 \end{array}$ | $\begin{array}{\|l\|} \hline 01 \\ 11 \end{array}$ | $\begin{array}{\|l\|} \hline 10 \\ 11 \end{array}$ | $\begin{array}{\|l\|} \hline 11 \\ 11 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\sim$ | 11 10 10 | 11 10 | 10 10 | $\begin{array}{\|l\|} \hline 11 \\ 10 \end{array}$ |
| - | $\begin{array}{\|l\|} \hline 00 \\ 01 \end{array}$ | $\begin{array}{\|l\|} \hline 01 \\ 01 \end{array}$ | $\begin{array}{\|l\|} \hline 10 \\ 01 \end{array}$ | $\begin{array}{\|l\|} \hline 11 \\ 01 \end{array}$ |
| $\bigcirc$ | 00 00 | 01 00 | $\begin{array}{\|l\|} \hline 10 \\ 00 \end{array}$ | $\begin{array}{\|l\|} \hline 11 \\ 00 \end{array}$ |


| m | 0101 | 0111 | 1101 | 1111 |
| :---: | :---: | :---: | :---: | :---: |
| $\sim$ | 0100 | 0110 | 1100 | 1110 |
| - | 0001 | 0011 | 1001 | 101 |
| $\bigcirc$ | 0000 | 0010 | 1000 | 1010 |
|  | 0 | 1 | 2 | 3 |

Interleave them to create one string

## Hilbert Curves (Step 3)

m
$\sim$

$\sim$$|$| 0101 | 0111 | 1101 | 1111 |
| :---: | :---: | :---: | :---: |
| 0100 | 0110 | 1100 | 1110 |
| 0001 | 0011 | 1001 | 1011 |
| 0000 | 0010 | 1000 | 1010 |
| 0 | 1 | 2 | 3 |


| $m$ | 0101 | 0111 | 1101 | 1111 |
| :---: | :---: | :---: | :---: | :---: |
| $\sim$ | 0100 | 0110 | 1100 | 1110 |
| - | 0001 | 0011 | 1001 | 1011 |
| $\bigcirc$ | 0000 | 0010 | 1000 | 1010 |
|  | 0 | 1 | 2 | 3 |

Divide the string into from left to right into 2-bit strings

## Hilbert Curves (Step 4)

| $\cdots$ | 0101 | 0111 | 1101 | 1111 |
| :---: | :---: | :---: | :---: | :---: |
| $\sim$ | 0100 | 0110 | 1100 | 1110 |
| - | 0001 | 0011 | 1001 | 1011 |
| $\bigcirc$ | 0000 | 0010 | 1000 | 1010 |
|  | 0 | 1 | 2 | 3 |



Assign decimal values: "00" as 0; "01" as 1; " 10 " as 3;
"11" as 2

## Hilbert Curves (Step 5)

$m$
$\sim$
$\sim$

$\sim$$|$| 11 | 12 | 21 | 22 |
| :--- | :--- | :--- | :--- |
| 10 | 13 | 20 | 23 |
| 01 | 02 | 31 | 32 |
| 00 | 03 | 30 | 33 |

$m$
$\sim$
$\sim$

$\sim$ | 11 | 12 | 21 | 22 |
| :--- | :--- | :--- | :--- |
| 10 | 13 | 20 | 23 |
| 03 | 02 | 31 | 30 |
| 00 | 01 | 32 | 33 |

If $j==0$ then switch every following occurrence of 1 to 3 and vice-versa

If $\mathrm{j}==3$ then switch every following occurrence of 0 to 2 and vice-versa

## Hilbert Curves (Step 6)



## Hilbert Curves (Step 6)

m
$\sim$

$\sim$ | 0101 | 0110 | 1001 | 1010 |
| :---: | :---: | :---: | :---: |
|  | 0100 | 0111 | 1000 |
| 0011 | 0010 | 1101 |  |
| 0000 | 0001 | 1110 | 1111 |
| 0 | 1 | 2 | 3 |


| m | 5 | 6 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| $\sim$ | 4 | 7 | 8 | 11 |
| - | 3 | 2 | 13 | 12 |
| 0 | 0 | 1 | 14 | 15 |
|  | 0 |  | 2 | 3 |

Concatenate and
Convert to Binary

## Hilbert Curves (Step 6)




- Hilbert-curve: $(0,0)(1,0)(1,1)(0,1)(0,2)(0,3)(1,3)(1,2)(2,2)(2,3)(3,3)(3,2)(3,1)(2,1)(2,0)$ $(3,0)$

Hilbert Curves (Step 6)



- Hilbert-curve: $(0,0)(1,0)(1,1)(0,1)(0,2)(0,3)(1,3)(1,2)(2,2)(2,3)(3,3)$ $(3,2)(3,1)(2,1)(2,0)(3,0)$


## Hilbert Curves Vs Z-Curves




## Hilbert- curve: Range Query




C
Hilbert-curver: $(0,0)(1,0)(1,1)(0,1)(0,2)(0,3)(1,3)(1,2)(2,2)(2,3)(3,3)(3,2)(3,1)(2,1)(2,0)(3,0)$


Need to pic min and max Hilbert curve values for this range!

## Hilbert Curves in larger spaces



Image source and more details at: http://www.bic.m ni.mcgill.ca/~mall ar/CS-
644B/hilbert.html

## Contemplating the Hillbert Curve Algo

- Step 1: Read in the n-bit binary representation of the $x$ and $y$ coordinates.
- Step 2: Interleave bits of the two binary numbers into one string
- Step3: Divide the string into from left to right into 2-bit strings
" Step4: Assign decimal values: "00" as 0; "01" as 1; "10" as 3; " 11 " as 2 and put into an array is the same order as the strings occurred.
- Step5: For each number j in the array
- If $\mathrm{j}==0$ then switch every following occurrence of 1 to 3 and vice-versa
- If $\mathrm{j}==3$ then switch every following occurrence of 0 to 2 and vice-versa
- Step6: Convert each number in the array to its binary representation (2-bit strings), concatenate from left to right and convert to decimal.


## Contemplating the Hilbert Curve Algo




## Output after Step 4

## Contemplating the Hilbert Curve Algo



## Contemplating the Hilbert Curve Algo

## Say we Skip Step 5 and Jump to step 6

$\sim$
$\sim$
$\sim$

$\sim$| 01 | 01 | 10 | 10 |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| 01 | 10 | 01 | 10 |  |  |  |  |  |
| 01 | 01 | 10 | 10 |  |  |  |  |  |
| 00 | 11 | 00 | 11 |  |  |  |  |  |
| 00 | 00 | 11 | 11 |  |  |  |  |  |
| 01 | 10 | 01 | 10 |  |  |  |  |  |
| 00 | 00 | 11 | 11 |  |  |  |  |  |
| 00 | 11 | 00 | 11 |  |  |  |  |  |
| 0 |  |  |  |  |  | 1 | 2 | 3 |



Step 6: We convert nums to binary, concatenate and then convert to decimal

## Contemplating the Hilbert Curve Algo

> Say we Skip Step 5 and Jump to step 6

| m | 01 01 | $\begin{aligned} & 01 \\ & 10 \end{aligned}$ | $\begin{aligned} & 10 \\ & 01 \end{aligned}$ | $\begin{aligned} & 10 \\ & 10 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\sim$ | 01 00 | $\begin{aligned} & 01 \\ & 11 \end{aligned}$ | $\begin{aligned} & 10 \\ & 00 \end{aligned}$ | $\begin{aligned} & 10 \\ & 11 \end{aligned}$ |
| - | 00 01 | $\begin{array}{\|l\|} \hline 00 \\ 10 \end{array}$ | $\begin{aligned} & 11 \\ & 01 \end{aligned}$ | $\begin{aligned} & 11 \\ & 10 \end{aligned}$ |
| $\bigcirc$ | 00 00 | 00 11 | $\begin{aligned} & 11 \\ & 00 \end{aligned}$ | $\begin{aligned} & \hline 11 \\ & 11 \end{aligned}$ |
| 2 |  |  |  |  |



Step 6: We convert nums to binary, concatenate and then convert to decimal

## Contemplating the Hilbert Curve Algo




Step 5 seems to be taking care of the rotation and the reflection of the basic shape inverted cup!!!

Addressing challenges of
2-Dimenions more directly

## Grid Files

Basic idea- Divide space into cells by a grid

- Store data in each cell in distinct disk page
- A directory structure needed
- Efficient for find, insert, range and nearest neighbor
- May have wastage of disk storage space
- Non-uniform data distribution over
 space ??


## Grid Files

## Refinement of basic idea into Grid Files

- Use non-uniform grids
- Linear scale store row and column boundaries
- Allow sharing of disk pages across grid cells



## Grid Files (insertion example)

- Capacity of bucket = 3

J. Nievergelt and H. Hinterberger. The Grid File: An Adaptable, Symmetric Multikey File Structure. ACM Transactions on Database Systems, Vol. 9, No. 1, March 1994


## Grid Files (insertion example)

- When the bucket overflows we split it.
- A new bucket is made.
- Records that lie in one half of the space are moved to the new bucket.

J. Nievergelt and H. Hinterberger. The Grid File: An Adaptable, Symmetric Multikey File Structure. ACM Transactions on Database Systems, Vol. 9, No. 1, March 1994


## Grid Files (insertion example)

- Bucket A overflows again.

J. Nievergelt and H. Hinterberger. The Grid File: An Adaptable, Symmetric Multikey File Structure. ACM Transactions on Database Systems, Vol. 9, No. 1, March 1994


## Grid Files (insertion example)

- Bucket A overflows again.

Very Imp: Splitting of A is full horizontal split, i.e., region of $B$ is also split. But B was not overflowing, so both buckets still point to B only
J. Nievergelt and H. Hinterberger. The Grid File: An Adaptable, Symmetric Multikey File Structure. ACM Transactions on Database Systems, Vol. 9, No. 1, March 1994

## Grid Files (insertion example)

- Bucket A overflows again.


## In Grid files, data space which are the buckets is different from the geographic spread of the data.


J. Nievergelt and H. Hinterberger. The Grid File: An Adaptable, Symmetric Multikey File Structure. ACM Transactions on Database Systems, Vol. 9, No. 1, March 1994

## Grid Files (insertion example)

- Bucket A overflows again.


## Splits in any dimension are made through and trough out. This makes the task of maintain linear scales easy


J. Nievergelt and H. Hinterberger. The Grid File: An Adaptable, Symmetric Multikey File Structure. ACM Transactions on Database Systems, Vol. 9, No. 1, March 1994

## Grid Files (insertion example)

- One more split.

J. Nievergelt and H. Hinterberger. The Grid File: An Adaptable, Symmetric Multikey File Structure. ACM Transactions on Database Systems, Vol. 9, No. 1, March 1994


## Grid Files (insertion example)

- One more split.
- Note that splits in any dimension are made through and trough.

J. Nievergelt and H. Hinterberger. The Grid File: An Adaptable, Symmetric Multikey File Structure. ACM Transactions on Database Systems, Vol. 9, No. 1, March 1994


## Grid Files (Another example)

- Assume Bucket size $=3$



## Grid Files (Another example)

- Assume Bucket size $=3$



## Grid Files (Another example)

- Assume Bucket size $=3$

Overflow! Create a new


## Grid Files (Another example)

- Assume Bucket size $=3$



## Grid Files (Another example)

- Assume Bucket size $=3$



## Grid Files (Another example)

- Assume Bucket size $=3$


Overflow! Create a new bucket; Split both scales and the bucket.

## Grid Files (Another example)

- Assume Bucket size $=3$



## Grid Files (Another example)

- Assume Bucket size $=3$



## Grid Files (Another example)

- Assume Bucket size $=3$


Overflow! Create a new bucket. Split bucket A.

## Grid Files (Another example)

- Assume Bucket size $=3$



## Grid Files (Another example)

- Assume Bucket size $=3$



## Grid Files (Another example)

- Assume Bucket size $=3$



## Grid Files (Splitting Policies)

- Splits:
- Can happen during insertion.
- Overflow of a bucket corresponding to a grid partition leads to a split.
- Can also happen if bucket containing records from several grid partition fills up.
- Splitting dimension can be changed alternatively.
- Splitting point may not always be the middle point, other algorithms are also possible.
J. Nievergelt and H. Hinterberger. The Grid File: An Adaptable, Symmetric Multikey File Structure. ACM Transactions on Database

Systems, Vol. 9, No. 1, March 1994

## Grid Files (Querying example)

- X-partitions (0,1000,1500,1750,1875,2000)
- Y-partitions (a,f,k, p,z).

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## Grid Files (Querying example)

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## Thoughts on <br> Precision and Recall of the initial step of this algorithm?


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## Grid Files (Merging Policies)

- Merging:
- Happens when data is being deleted.
- Buckets may be merged in case of underflow.
- Multiple policies can be developed for merging.
- Details beyond the scope of this course.
- Interested readers can refer the paper for details.

